

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

5-6

NATURE OF THE ACTIVITIES SUGGESTED HERE

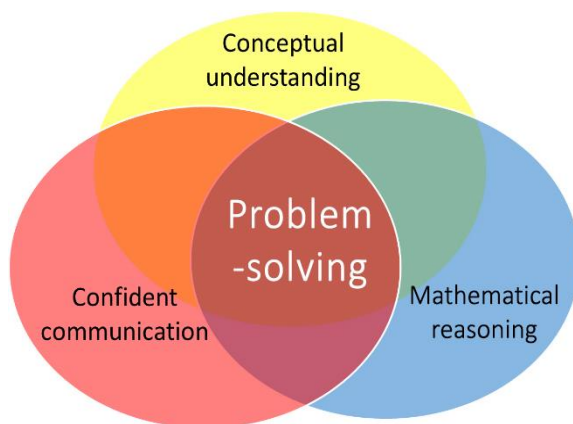
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

18. Proportionality and Percentages

Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'.

Write percentages as a fraction with denominator 100.

Solve problems which require knowing percentage and equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

A very common real need to use percentages is to compare prices before and after discounts, price reductions or price rises. In this activity the Meena and Charlie work for *Barry's Bikes* and their job is to update the prices on the

Catalogue changes Children in pairs, working individually but with peer-assessment. They will need:

- An interesting page from a *real* catalogue is best, but for the purposes of this description here is a fictitious example (see worksheets):
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<i>Barry's Bikes Catalogue</i>		<i>Accessories page</i>	
A. LED light set	\$20.99	F. Bike helmet	\$9.00
B. Twin mudguards	\$17.00	G. 'D' lock	\$16.99
C. Tyre pump	\$8.50	H. Cycle computer	\$9.00
D. Rack	\$23.95	I. Basket	\$11.99
E. Gel cycle seat	\$25.00	J. Puncture repair kit	\$2.49

Barry has reviewed his sales and has decided that some items are not selling well, so in the next catalogue, these will be reduced. Other items are very popular or have been replaced by better quality products and Barry wants to increase their prices.

First, model/revise the calculation of an example price reduction and/or increase. For example, if a puncture repair kit increases in price by 20%, that would be 50c increase (typically rounded up – see below*), so the new price would be \$2.99.

Do the children know that, if the whole value of something is 100%, then to find:

- 10% is $\frac{1}{10}$, so divide by 10?
- 1% is $\frac{1}{100}$, so divide by 100 (or find $\frac{1}{10}$, and then $\frac{1}{10}$ of that)?
- 5% is $\frac{1}{20}$, so halve 10%?
- 20% is $\frac{1}{5}$, so divide by 5 or double 10%?
- 25% is $\frac{1}{4}$, so divide by 4 or halve and halve again?

Do they see that it is often easier to calculate the new price by using the percentage to add to or subtract from the original amount, rather than multiply or divide by a factor?

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accessories page of the shop's catalogue.

Meena and Charlie have the following changes to make for the catalogue and need to work out the new prices, for example:

- Items C and J increased by 10% (C = \$9.35; J = \$2.74);
- Items H and I reduced by 5% (H = \$8.55; I = \$11.39);
- Items B and D increased by 25% (B = \$21.25; J = \$29.94);
- Items A and E reduced by 20% (A = \$16.79; E = \$20.00);
- Items F and I increased by 40% (F = \$20.30; I = \$16.79).

The children estimate, calculate and compare their results to check and agree the figures they will present to Barry.

*In Chapter 18, ad hoc methods are used to build up 'awkward' percentages such as 17% from its component parts: 10% + 5% + 1% + 1%. This is fine if the price is a whole number of pounds, or for estimating the result. However, for *realistic* prices in a catalogue including odd numbers of cents, this approach will yield some inaccuracy if fractions of pennies are rounded at each intermediate step, so some tolerance may be needed if this is done. Therefore when setting problems for the children to do on *realistic* prices without a calculator, it is wise to limit these to one or two step calculations on a single number to limit the number of intermediate steps at which amounts may be rounded.

For higher attainers, children can work with further decimal places (fractions of cents) until the calculation is complete before rounding just the result to nearest cent.

Hundreds

Tens

Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

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WORKSHEETS FOR UPPER PRIMARY

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

WORKSHEETS FOR UPPER PRIMARY

5-6

GO OFY WAYS TO MAKE 1!

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

CATALOGUE CHANGES

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SPORTS MATHS

<i>Team:</i>										
Event Name:										
Child's Name										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

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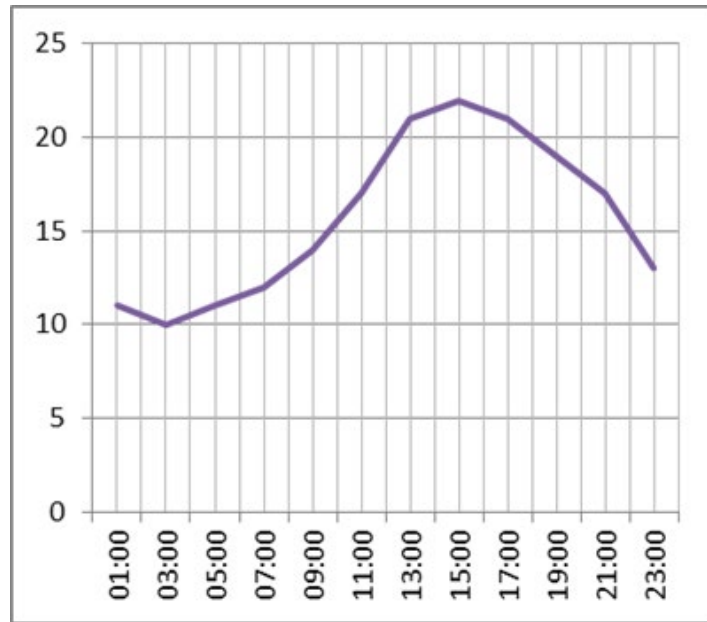
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<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

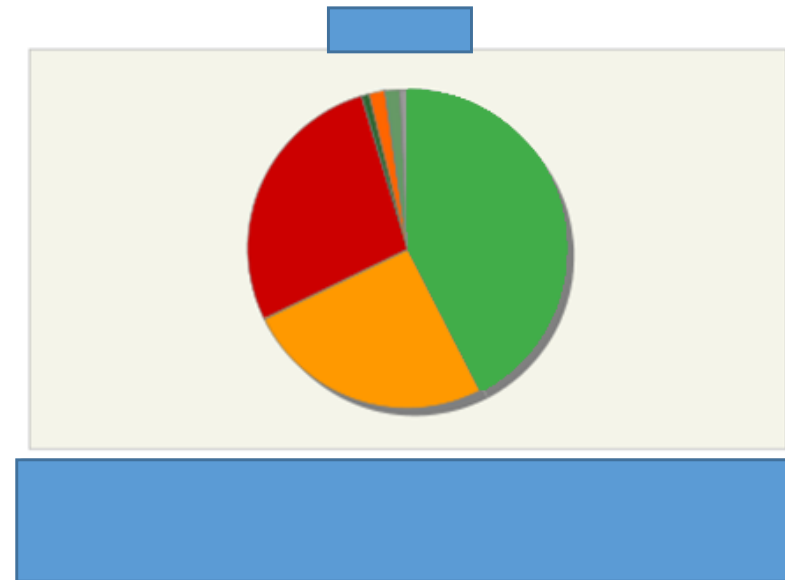
<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

DATA DETECTIVES

A. Line Graph

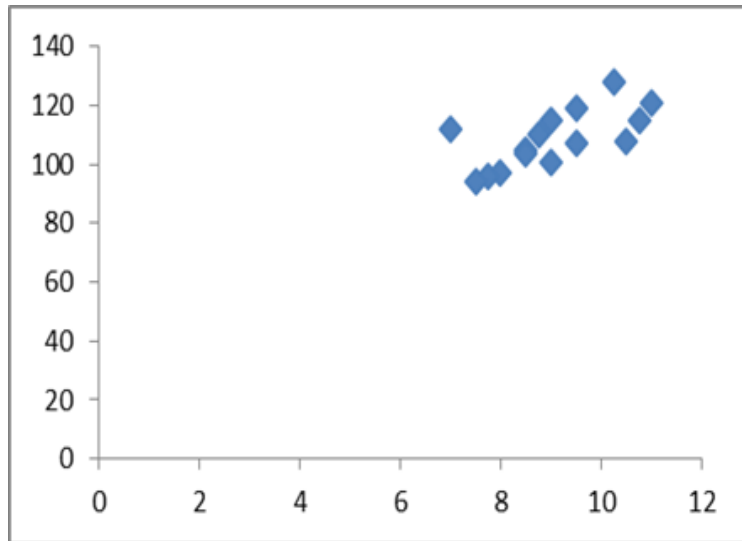


B. Pie Chart

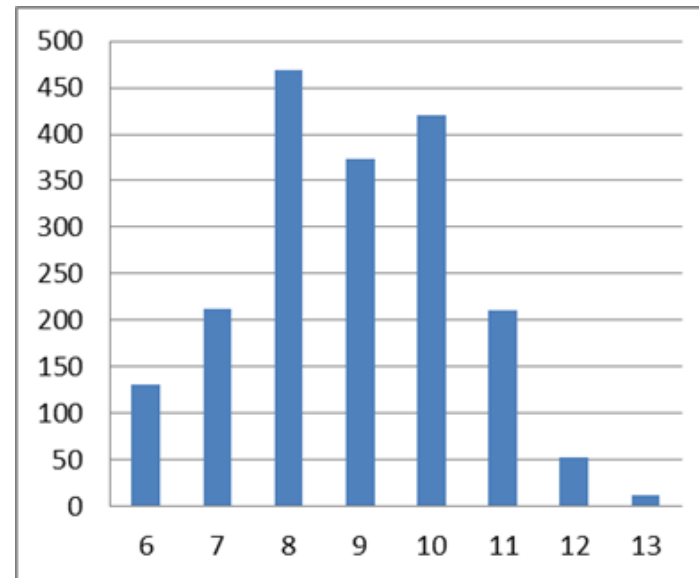


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



WORKSHEETS FOR UPPER PRIMARY

TV PROGRAMMES

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	