PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

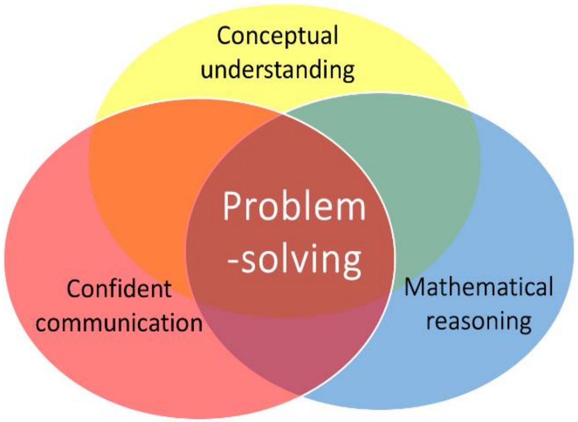
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery,* the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*. More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

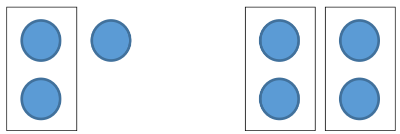
Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

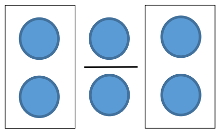
Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

19. Algebraic Reasoning **Recognise odd and even numbers.** **Understand the pattern for the outcomes of their additions.** It is very helpful in developing their understanding of algebra that they look for *generalisations*. Here, children explore the addition of numbers to determine the expectations of combining odd and/or even numbers.	**Odds and evens** In pairs. Children explore individually and compare their findings with one another. They will need: • Number cards from 1 to 10; • Counters, or *numicon* templates; • Prepared tables to record whether odd/even (see worksheets). First establish/revise the odd/even property of numbers from 1 to 10. One basic visual representation to split the number of counters into two equal lines (halves) then if there is the same number in each line after this, the number of counters was *even*. If there is one more in one pile than the other, then the number of counters was odd. Probably a more helpful way of testing whether a number is odd or even is to set out the counters in groups of 2. If there is a whole number of 2s without a counter 'left over', the number is even. If there is 1 counter left over, the number is odd. The counters can be arranged to show this. This is naturally more powerfully demonstrated by taking a *numicon* template for each number and then attempting to place a series of templates for 2 along the top of the number. Any number which is completely covered by '2s' is *even*, while any which shows an uncovered '1' from the template below is *odd*. For example:  3 is *odd* 4 is *even*	Do the children securely distinguish the property ***odd*** from ***even***? For example, do they recognise that evens have the property of 'being shared equally between 2'? Can children describe a rule in simple sentences? For example: 'When you add an *odd* number to another *odd* number, you will get an *even* number. When you add an *odd* number to an *even* number you always get an *odd* number.'

For each number Emily and Luke count out that number of counters, and see whether it is odd or even. They can then write the number under the appropriate heading in the table:

Odd	Even
1, 3, 5, 7, 9	2, 4, 6, 8, 10

Next ask Emily and Luke to see what happens when they combine different numbers by adding them together. Which additions give them a new *odd* number, which add together to make an *even* number? For example 3 + 3?



Again, this is something which is easy to demonstrate if the counters are grouped in '2s', or by using *numicon* templates, where the '1s' of two odd numbers combine by interlocking to make another '2'. Ask the children to write the additions under the appropriate table headings:

Odd	Even
1 + 2 = 3 2 + 3 = 5 3 + 4 = 7	1 + 1 = 2 2 + 2 = 4 1 + 3 = 4 3 + 3 = 6

Once the children have completed a number of additions can they see any rules for always getting an *odd* number, or always getting an *even* number?

To take it further, what happens when they combine three numbers? Can they begin to explain why?

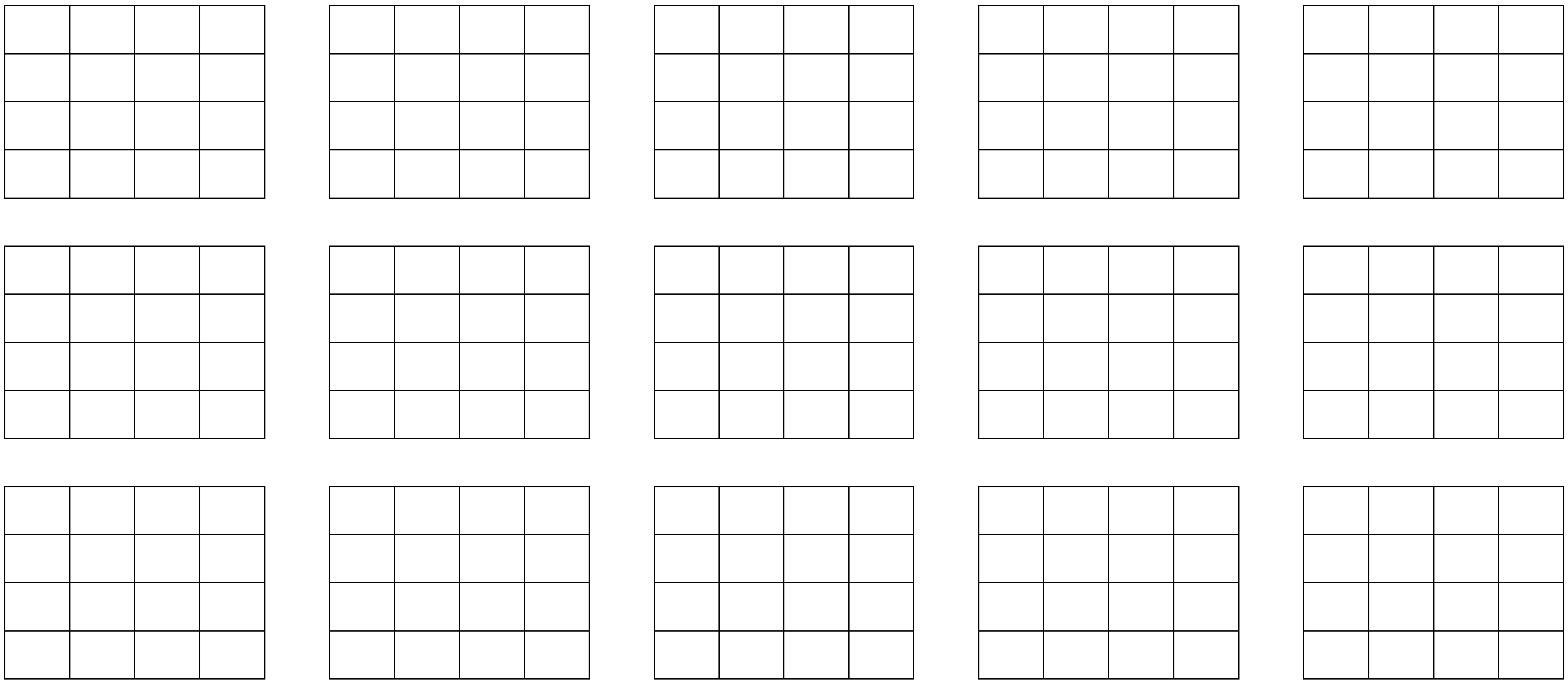
Hundreds	Tens	Ones

WORKSHEETS FOR LOWER PRIMARY

1-2

SEEING SQUARES

Cut into separate grids – 1 for each child



100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Colour:		
	in every	
	in every	
	in every	

Colour:		
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ODDS AND EVENS

Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

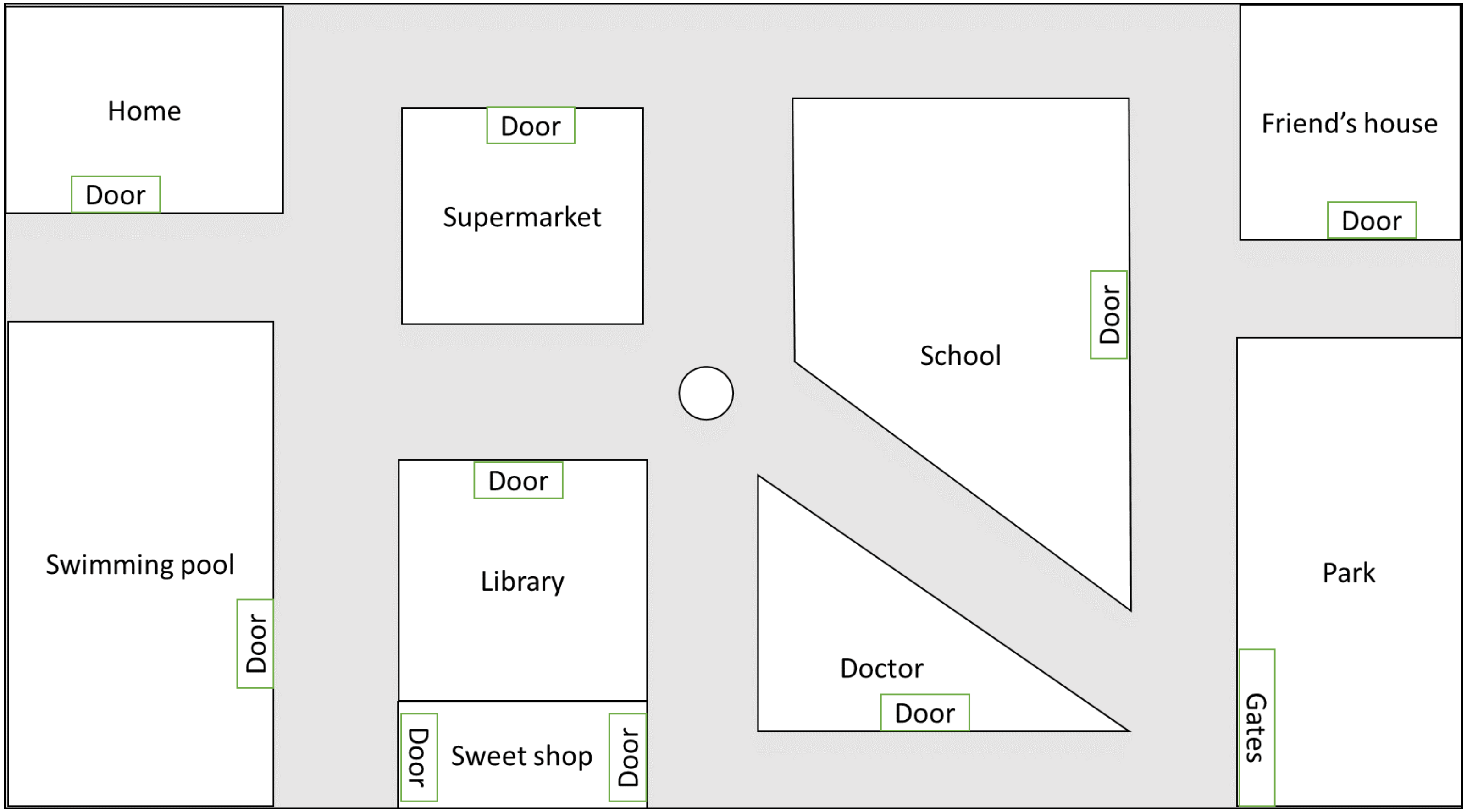
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

ROBOTS


Home
Door
Door
Supermarket
Friend's house
Door
Door
School
Swimming pool
Door
Door
Library
Park
Doctor
Door
Gates
Door
Sweet shop
Door

SHAPE SORTER

TRAFFIC SURVEY

Vehicle	*Tally*	*Total*

Vehicle	*Tally*	*Total*