PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

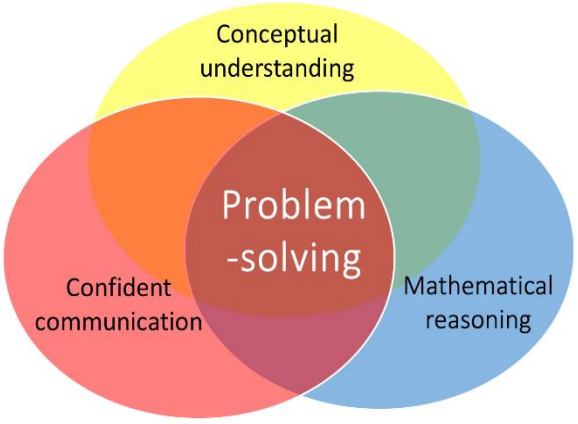
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery,* the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*. More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

20. Coordinates and Linear Relationships

Understand the use of simple row and column labelling to identify position in 2 dimensions.

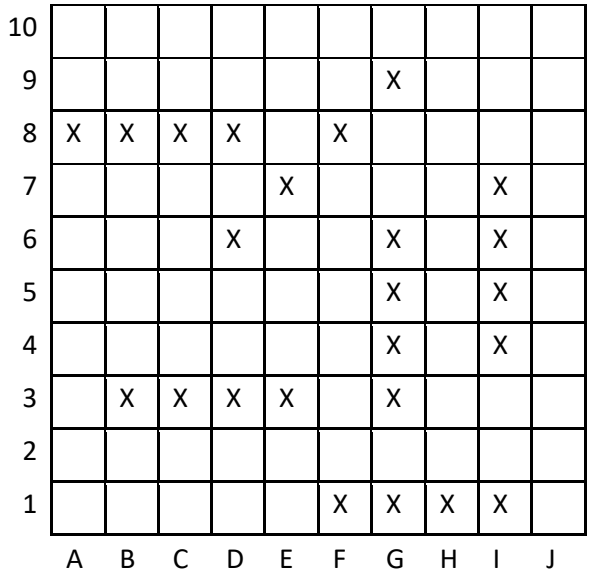
In this activity, children explore position, identifying a row and column reference to locate a square on the paper. While this version does not use a frame of true co-ordinate references (which locate points, rather than whole squares), it is a helpful way for younger children to identify position in 2 dimensions. Row and column references are also used on some maps, for example, *A-Z street maps*.

Simple battleships Children play in pairs. Each pair will need:

- Sheets of squared paper, or prepared grids – one each (there is a worksheet available);
- A paper screen to enable them to hide their grids from one another (see below how to make this).

The activity here is the traditional paper game of *battleships*.

Ships are set out as crosses in a line of whole squares, so there is no debate about whether a targeted square is part of a battleship, e.g.:



First of all, give out the prepared grids, or show the class how to mark out a large square grid on their squared paper, comprising 10 rows and 10 columns. Label the squares along the ***horizontal*** base with the letters A–J, and the squares up the left ***vertical*** edge with the numbers 1–10.

Show how to locate any single square by identifying the combination of its row and column, for example, D6, F9, and practise this with the children. Emphasise that we give the horizontal location first when describing any row and column pair (some remember this as '*along* the hall ***then*** *up* the stairs'). This will be an important order to retain when making the transition to point co-ordinates later.

Now explain that this grid is an area of the ocean, where battleships are out 'on manoeuvres'. Show how children may place their battleships. Each battleship is represented by a single line of four *crossed* squares, positioned horizontally, vertically or diagonally. Invite the children to fire a

Do the children understand the terms ***horizontal*** and ***vertical?***

Do the children identify an intended square correctly?

Do the children recognise the relationship of the identification for each square with its neighbouring squares?

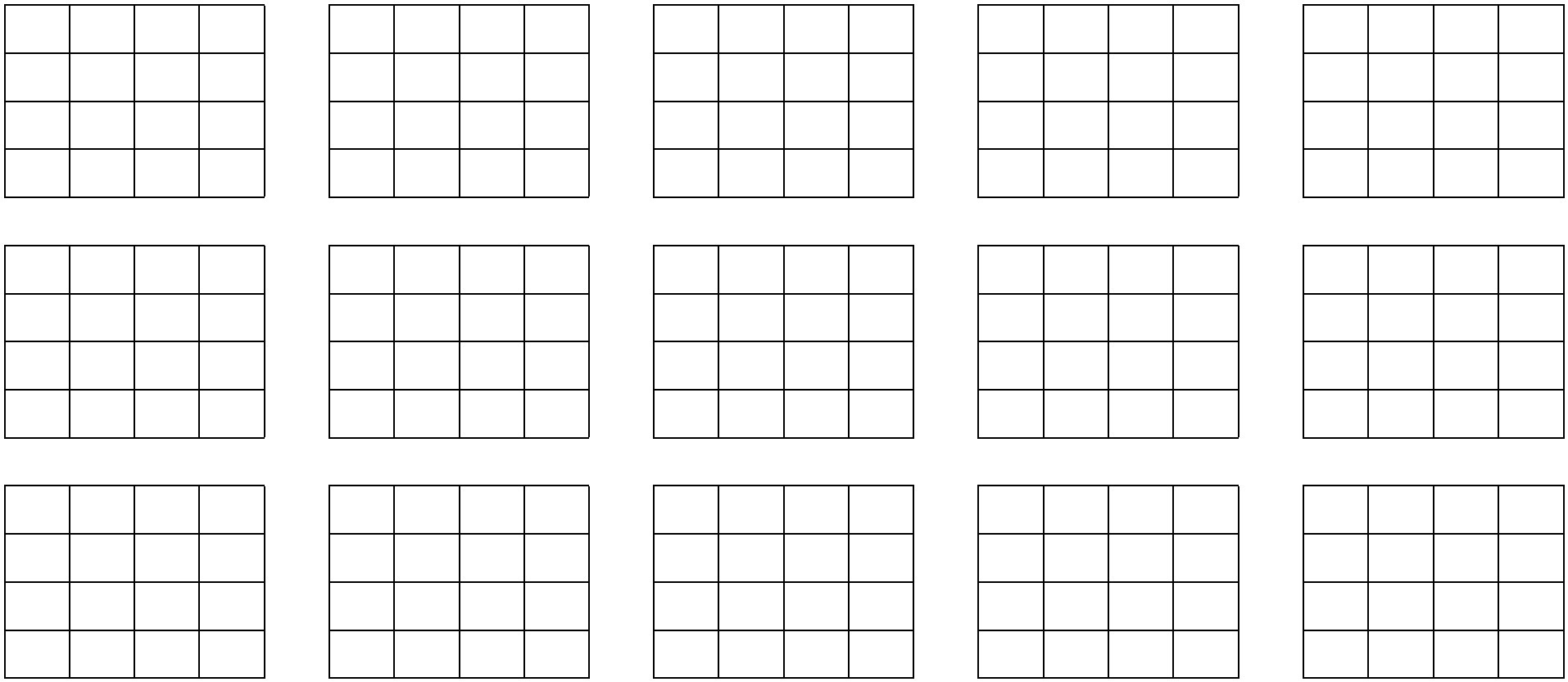
Do they associate the *rightward* direction in labelling *alphabetically* along the ***horizontal*** axis, and the *upward* direction for labelling *numerically* along the ***vertical*** axis?

	four *crossed* squares, positioned horizontally, vertically or diagonally. Invite the children to fire a shot at one of your battleships, by naming a square it is covering. A 'hit' on any crossed square destroys the whole of that ship. Luke and Emily hide their grids from one another behind the paper screen and mark out the same number of battleships. Then each takes it in turn to 'fire' a row and column location to the other: • For example, if Luke goes first he may call 'C3'. Emily circles the location C3 on her grid and tells Luke whether it is a 'hit' or a 'miss'; • It is important that they answer truthfully whether a hit has been made upon one of their ships. It is useful that they each write a list of locations they have fired at, to check any disagreement at the end of the game; • The winner is the first to score *at least* one hit on *every* one of their opponent's ships. When this happens, they take down the screen, and then check every shot they have recorded to confirm the result. To make the game last longer, include ships of a shorter length. It is very hard to hit a ship whose length is just one square! Alternatively, use smaller grids to make the game shorter or easier.	

Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Colour:		
	in every	
	in every	
	in every	

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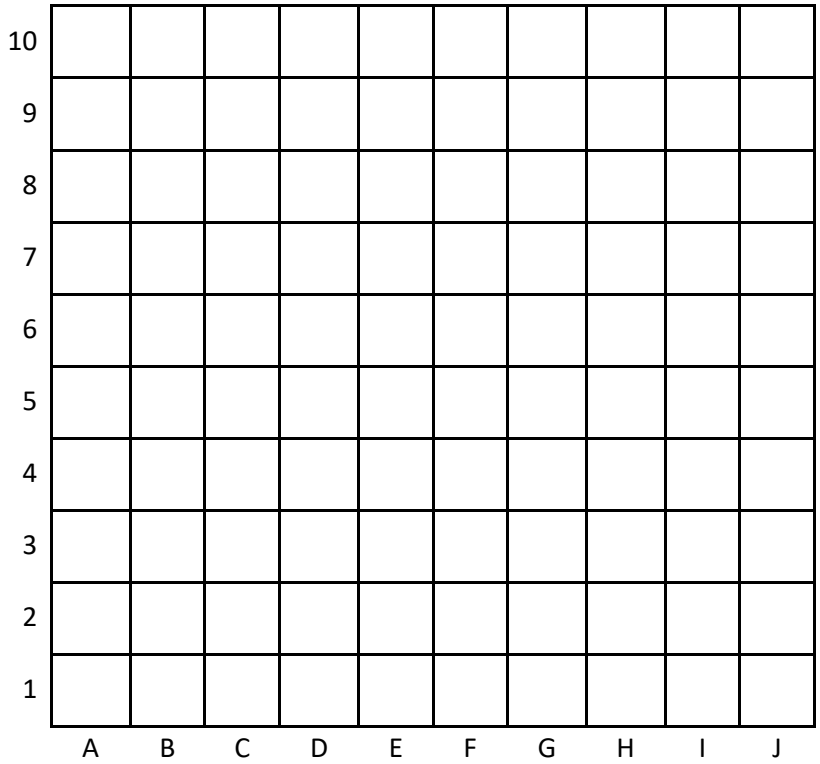
ODDS AND EVENS

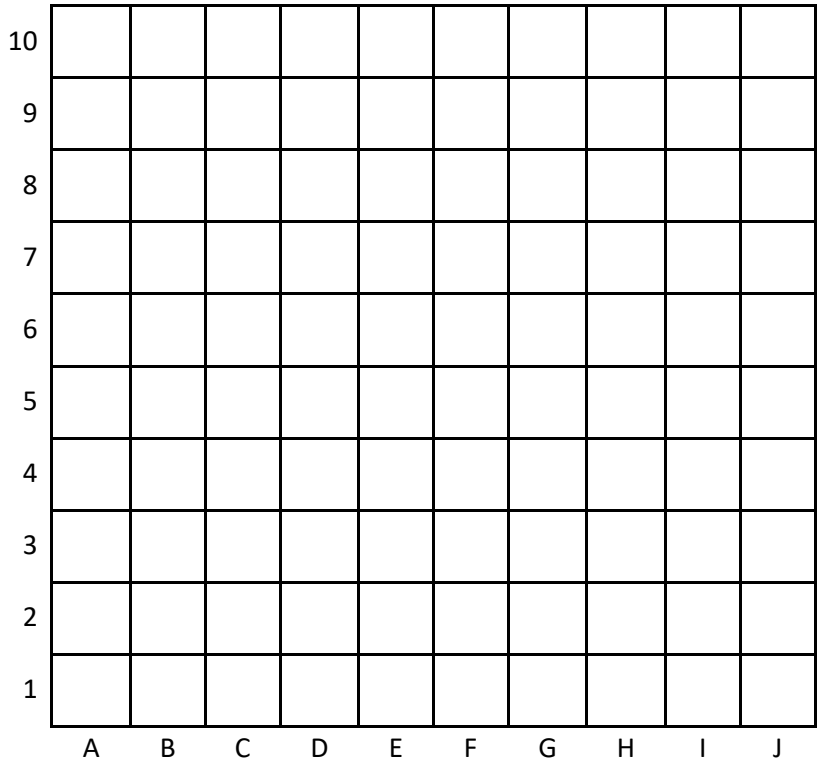
Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

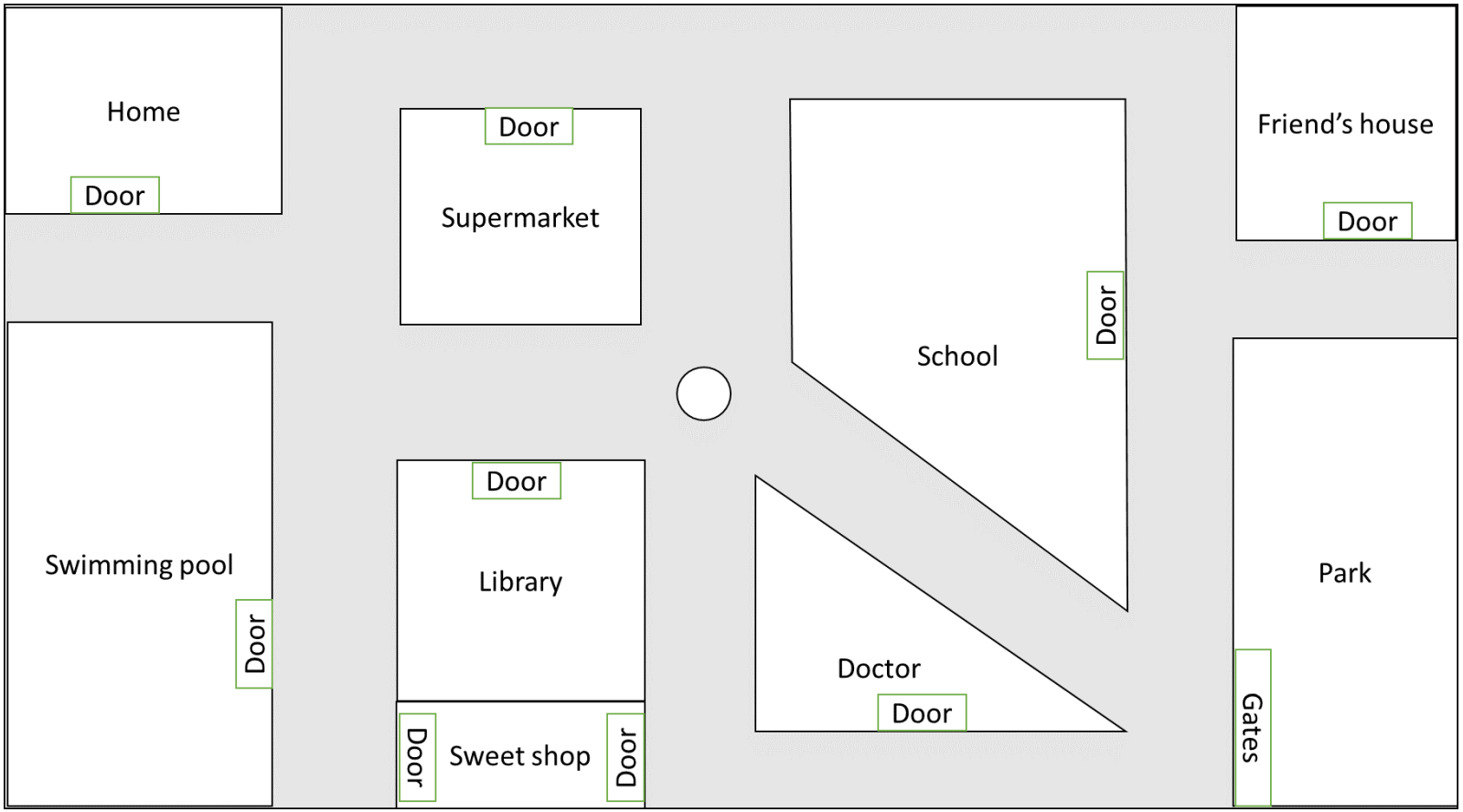


List of squares I have fired at:



List of squares I have fired at:

ROBOTS


Home
Door
Door
Supermarket
School
Door
Friend's house
Door
Swimming pool
Door
Door
Library
Park
Doctor
Door
Door
Sweet shop
Door
Gates

SHAPE SORTER

TRAFFIC SURVEY

Vehicle	*Tally*	*Total*

Vehicle	*Tally*	*Total*