

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

3-4

NATURE OF THE ACTIVITIES SUGGESTED HERE

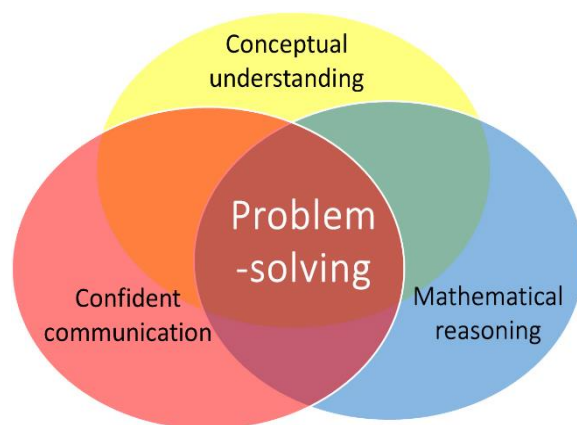
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner* and the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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<p>20. Coordinates and Linear Relationships</p> <p>Understand the use of co-ordinates to identify a point location in the 1st quadrant (positive values).</p> <p>This is to give children practical experience referring to 2-dimensional co-ordinates to locate <i>points</i>, rather than <i>squares</i>. The children will also begin to see how co-ordinates can be a powerful tool when providing accurate sequences of instructions.</p>	<p>Co-ordinated shapes Children work in pairs. They will need:</p> <ul style="list-style-type: none">• Sheets of squared paper• A paper screen to enable them to hide their grids from one another (see below how to make this). <p>On graphs and on geographers' maps, two numbered co-ordinates identify a single <i>point</i> where a (virtual) line perpendicular to a position along the horizontal axis intersects with another (virtual) line perpendicular to a position along the vertical axis.</p> <p>Project a squared grid upon which you have drawn the horizontal and vertical axes. Show how each of the axes has been labelled from zero where the other axis crosses it, in positive whole numbers at each regular point where a grid line crosses the axis. (Refer to the 1st quadrant only at this age.)</p> <p>Demonstrate how we can use this to identify any point where grid lines cross – from the origin (0, 0), showing how these co-ordinates expressed as a pair of numbers in this way represent the numbers of the gridlines to follow perpendicular to each axis. (Remember: '<i>along the hall, then up the stairs</i>'.)</p> <p>Mark some points on your display and ask the class to tell you the co-ordinates. Explore the values they give and write on the correct ones. Ask children to come up and mark on your display the points identified by given pairs of co-ordinates. When this appears to be secure the children can begin their independent activity.</p> <p>Shelley and Rohan draw and number the horizontal and vertical axes on their own sheets of squared paper.</p>	<p>Do the children recognise that the co-ordinates identify the intersection of the horizontal and vertical gridlines?</p> <p>Do the children identify and interpret the co-ordinates correctly, and in the correct order to create each shape? Do they understand and correctly use the key vocabulary: horizontal, vertical, axis, axes, origin, perpendicular, co-ordinates?</p>
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Each hiding their sheet behind the the paper screen, they then draw a series of different, irregular 2D shapes, **whose vertices are grid points on the squared paper**: for example, Shelley draws a quadrilateral, a pentagon and a hexagon. The shapes can overlap, so the children use different coloured pencils to see them more clearly. As they draw their shapes, each child writes the co-ordinates for each successive point in a list on another sheet of paper.

When they have completed the shapes and listed the co-ordinates for each one, Shelley and Rohan exchange their lists of co-ordinates, and attempt to recreate each other's designs. Encourage the children to be imaginative in the design of their shapes. For example, Shelley's design and lists of co-ordinates could be:

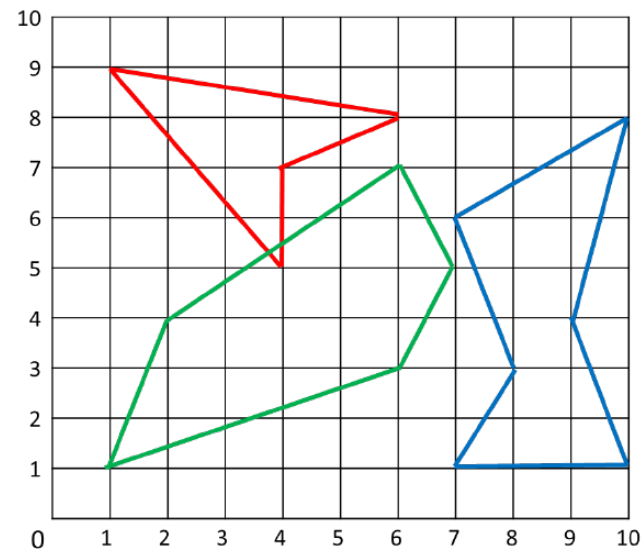
Quadrilateral: (1, 9), (6, 8), (4, 7) & (4, 5);

Pentagon: (1, 1), (2, 4), (6, 7), (7, 5) & (6, 3);

Hexagon: (7, 1), (8, 3), (7, 6), (10, 8), (9, 4) & (10, 1);

Afterwards they compare their reproductions with the original designs made by their partner. If there are any mistakes, is this because of an inaccuracy in the co-ordinates given or their interpretation?

The activity can be extended by drawing further shapes, or children could use a smaller grid and simpler shapes, so that there are fewer co-ordinates and are easier to interpret.



Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					
	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

ROUNDING TO ESTIMATE

<i>The Cosy Café</i>		
<u>Savoury</u>		
Sandwich	\$1.95	Baguette \$2.75
Tortilla wrap	\$1.45	Pizza slice \$2.10
<u>Sweet</u>		
Scone and jam	\$2.10	Cake \$2.90
Shortbread	\$1.25	Tiffin \$1.75
<u>Drinks</u>		
Squash	95c	Juice \$1.35
Tea	\$1.65	Coffee \$2.15

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ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

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PROBABLE PROPORTIONS

<i>Colour</i>	<i>Tally</i>	<i>Total</i>

<i>Colour</i>	<i>Tally</i>	<i>Total</i>