PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

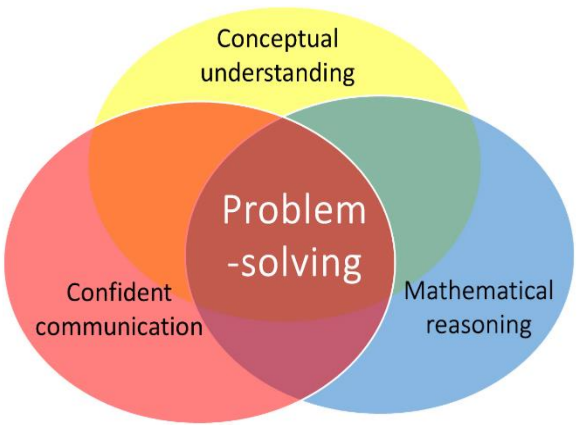
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of ***mastery***, the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner* and the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

22. Perimeter, Area and Volume **To understand that it is possible to produce different areas within the same perimeter.** **To understand that for rectangles with the same perimeter a *square* encloses the largest area.** This is a simple activity for children to explore the *(absence of a)* direct relationship between perimeter and area.	<u>**Free range area**</u> Children work in pairs, to discuss and compare findings. They will need: • Squared paper (1 cm^2). Explain that the school is considering keeping some chickens. If they do, the chickens will need to be housed and range freely within a penned, ***rectangular*** area. A local garden centre may supply, for free, the chicken wire to fence the pen. Display a grid pattern for the squared paper and explain that for your planning you will work with a scale that 1 grid square (1 cm^2) represents 1 square metre (1 m long × 1 m wide). If the garden centre provided a 12 m roll of chicken wire fencing to fence the perimeter, what different rectangular areas could be made, if we always measure the sides in whole metres? Which would be the largest area? Draw these with the children: show that if the whole perimeter is a maximum of 12 m, the pen could be 1 × 5 squares, 2 × 4 squares or 3 × 3 squares. How many squares comprise the area of each of these rectangular pens? Now ask Shelley and Rohan to use the squared paper draw the different rectangular pens they could enclose with a perimeter of 20 m of chicken wire, and the area for each one. Which pen would give the chickens the largest area to roam? Repeat with other perimeter values, such as 24 m and 28 m. What conclusions can Shelley and Rohan make about how to create the largest area? What would happen if the perimeter was 30 m? Ensure that children work with lengths of sides which are in whole metres. Compare and discuss findings as a whole class.	Do the children understand the difference between area and perimeter? Do the children see that they are drawing plan views of the pens (i.e. views from above)? Do they interpret the scale of 1 cm : 1 m correctly? Do they realise that the largest area for rectangle is when it is a square? Do they see that the more equal are the lengths of all the sides, the larger the area that can be enclosed for the same perimeter?

Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla
Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

ROUNDING TO ESTIMATE

The Cosy Café

Savoury			
Sandwich	$1.95	Baguette	$2.75
Tortilla wrap	$1.45	Pizza slice	$2.10
Sweet			
Scone and jam	$2.10	Cake	$2.90
Shortbread	$1.25	Tiffin	$1.75
Drinks			
Squash	95c	Juice	$1.35
Tea	$1.65	Coffee	$2.15

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ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

PROBABLE PROPORTIONS

Colour	*Tally*	*Total*

Colour	*Tally*	*Total*