PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

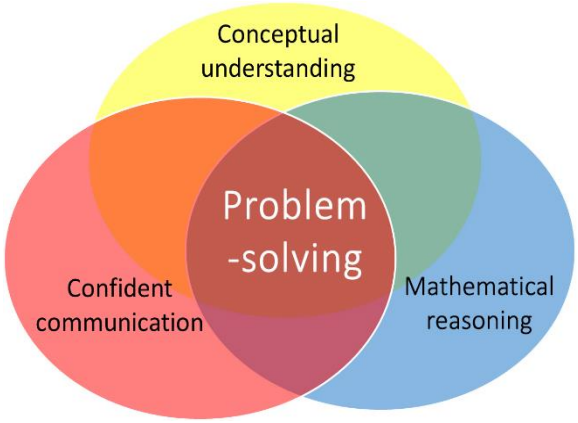
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia,including Bloom's* theories of ***mastery,*** the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

22. Perimeter, Area and Volume **To use and calculate areas for real purposes.** When painting a room, the liquid volume of paint to buy depends on the area to be painted, the number of coats to fully colour the walls, and the coverage property of the chosen paint, usually given on the tin. The coverage is usually described in terms of m^2/litre of paint.	**Painting wall areas** In groups of 4, to measure, calculate and check their findings. They will need: • Metre sticks or tape measures; • Recording chart (see worksheets). Explain that you think it is time the classroom walls were repainted (or some other part of the school). The children can help by working out how much paint the school needs to buy. Tell them we need to know the size of the area we wish to paint. Ask the children to remind you how to calculate the area of a rectangle and revise this if needed. Explain this is fine if we only need to paint one rectangular wall, but what should we do if we want to paint all the visible parts of the walls–ignoring cupboards, display boards, doors and other covered parts which do not need to be painted the same colour as the plastered walls? Help the children to see that the total area to be painted comprises a number of smaller areas and that these can be divided into rectangles. Charlie, Meena, Alexi and Woljca set off to measure the length and height of different parts of the walls to be painted. As they do so they complete the group's recording chart:	Do the children understand the idea of 'coverage' in terms of area per liquid volume of paint? Do the children divide the areas to be painted into a reasonable distribution of rectangles? Do they measure and calculate the areas of each of these rectangles correctly? Do they add all the areas together to find the total area needed to be covered?

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Back of classroom	6 m	2.5 m	15 m^2
Above the big display board	8 m	1 m	8 m^2
Below the big display board			
Right of big display board			
Above the fixed cupboards			
Total area to be painted:			

Provide the children with the height of the walls, so that they can work out the actual height of each area to paint by measuring and subtracting the heights of cupboards, boards etc.,

Do they take account of the number of coats needed and multiply the area accordingly?

	which are not to be painted. This will save the children needing to climb up to reach unsafe parts of the walls. Decide the precision necessary for the measurements and whether calculators may be used or children are to practise informal or formal written methods of multiplication. In real life, measurements of length could typically be rounded up/down to the nearest 1 m or 0.5 m. When they have measured all the smaller areas they can then calculate the paint to buy. For example: • What is the total area that needs to be painted? • If three coats of paint are needed to fully colour the walls, how much is the area that has to be covered if we count the area being painted three times? • If the coverage of a typical emulsion paint is 16 m^2/litre and the paint is sold in 5-litre, 2.5-litre and 1-litre tins, how many of each size should we buy? (It is more economical to buy paint in larger volumes). • How much would the paint cost, if the prices of these tins are: 5 litres for $29.50, 2.5 litres for $17 and 1 litre for $11?	

Hundreds	Tens	Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

GO OFY WAYS TO MAKE 1!

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

CATALOGUE CHANGES

Barry's Bikes Catalogue		Accessories page	
A. LED light set	$ 20.59	F. Bike helmet	$ 14.57
B. Twin mudguards	$ 16.21	G. 'D' lock	$ 16.9
C. Tyre pump	$ 8.34	H. Cycle computer	$ 9.35
D. Rack	$ 23.98	I. Basket	$ 11.93
E. Gel cycle seat	$ 25.89	J. Puncture repair kit	$ 2.49

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SPORTS MATHS

Team: **Event Name:** **Child's Name**										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

Shape	Angles	Sum of angles

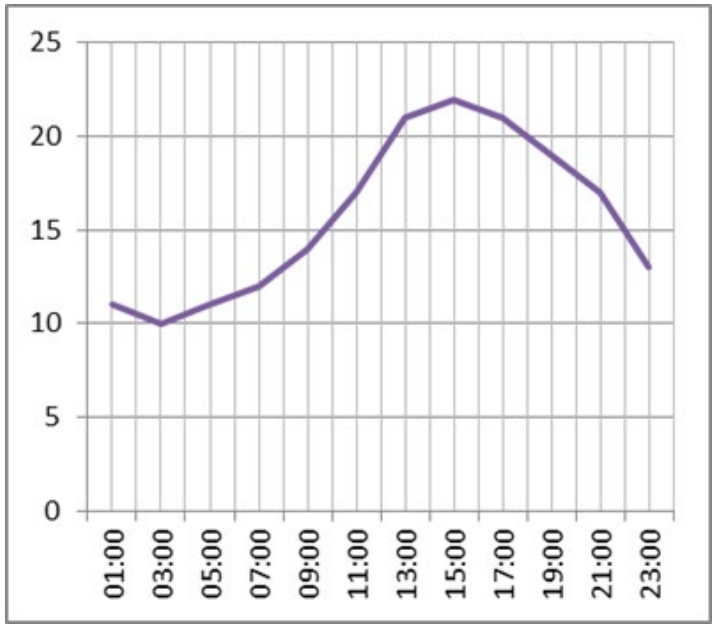
THE LIFE OF PI

object	diameter (cm)	circum-ference (cm)	Possible relationship?

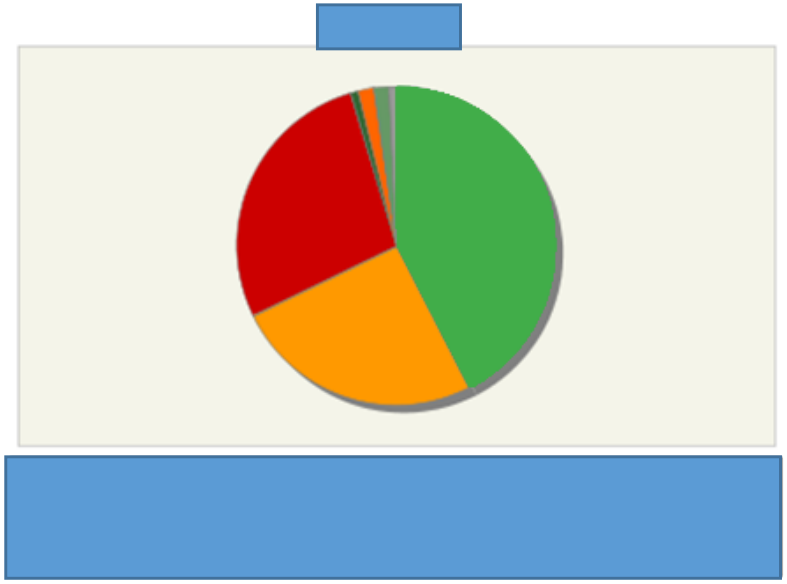
object	diameter (cm)	circum-ference (cm)	Possible relationship?

DATA DETECTIVES

A. Line Graph

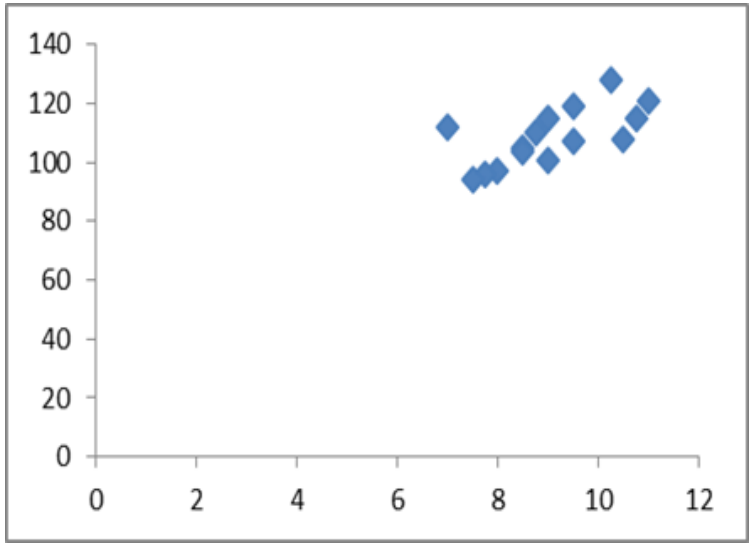


B. Pie Chart

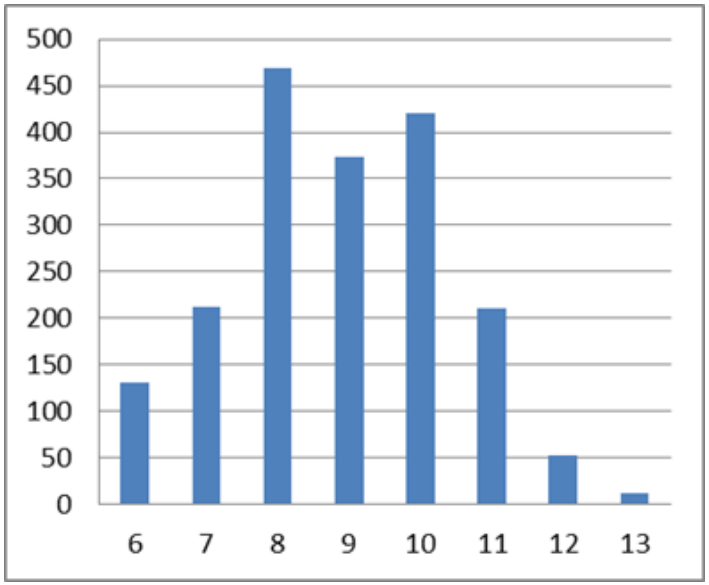


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



TV PROGRAMMES

Programme length	Frequency
Total:	

Programme length	Frequency
Total:	