

# PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

## NATURE OF THE ACTIVITIES SUGGESTED HERE

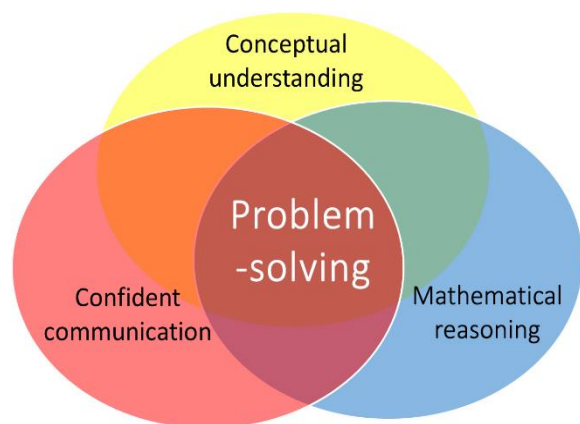
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA\* and TIMSS\* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's\* theories of *Mastery*, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner\*, the **realistic mathematics education** of Freudenthal\*. More recently, Lo's\* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury\*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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## NATURE OF THE ACTIVITIES SUGGESTED HERE

In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit [www.MathematicsMastered.org](http://www.MathematicsMastered.org)

### \*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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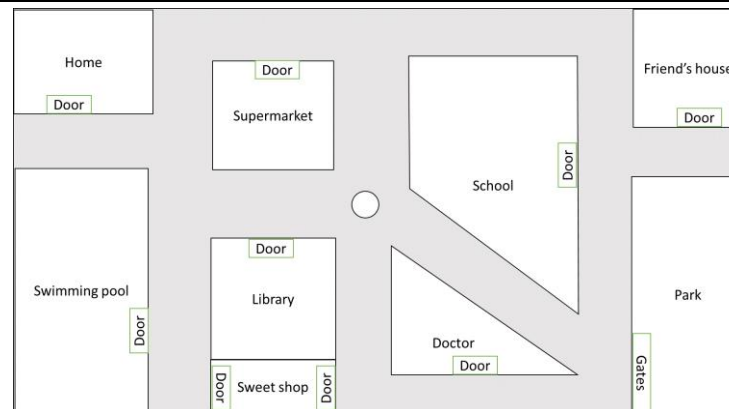
<p><b>23. Angle</b></p> <p><b>Developing the understanding of angle as a dynamic measure of a change in direction.</b></p> <p><b>Use the vocabulary: <i>whole, half, quarter turn, clockwise and anticlockwise.</i></b></p> <p>The ‘human robot’ and controllable toys are very engaging, practical ways to explore position and direction. They are therefore a helpful way to explore the dynamic aspect of angle.</p> <p>When discussing a change in direction, it is helpful to use the term <b>angle</b>, even though we are describing this in terms of parts of a <b>turn</b>. It helps children to begin to understand the concept of angle as a dynamic measure of a change in direction.</p>	<p><b>Robots</b> In groups of 2–4 children. Children will need:</p> <ul style="list-style-type: none"><li>• (Freestanding) controllable moving toy such as a ‘bee-bot’;</li><li>• Large sheet of sugar paper with a simple village road plan drawn upon it, suitable to be traversed by the ‘bee-bot’ (or whatever is used). See example in worksheets, which could be enlarged if you have a suitable copier.</li></ul> <p>Begin with the whole class standing practising turning in response to a demonstration and then by instruction alone, to teach/reinforce the terms for changing direction: <b>whole, half, quarter turn, clockwise</b> and <b>anti-clockwise</b>.</p> <p>Then appoint a child to become a ‘human robot’, and explain that the robot only understands certain commands – the instructions for changing direction, and the commands forward and back a given &lt;number&gt; of steps. Humorously test some trial instructions and non-instructions suggested by individual children in the class, reminding the ‘robot’ that unless the instruction is given in the correct form, they will have to ignore it. As a class provide instructions to direct the ‘robot’ to another part of the classroom.</p> <p>The children then carry this out in their groups. Emily, Luke, Kasia and Nathan each take an opportunity to become the ‘robot’, and the others give instructions to direct the robot to a place in the classroom (avoiding other ‘robots’ en route!).</p>	<p>Do children understand that <b>angle</b> refers to a concept of dynamic change in the direction of a line of travel,</p> <p>Do they understand angle can be measured simply to begin with in terms of <b>whole, half and quarter turns</b>, together with the orientation of the turn – <b>clockwise</b> or <b>anti-clockwise</b>?</p> <p>Can the children visualise the forward view from the position of the robot, whether human or toy? This is necessary in order to give the direction commands accurately to the robot.</p>
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One group can use the controllable toy to move it along the roads to different places on the village floor plan.

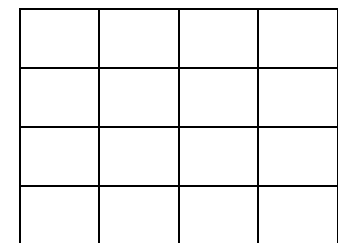
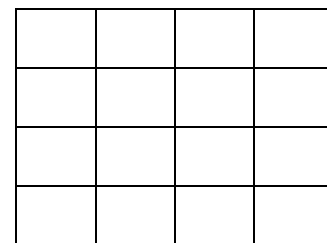
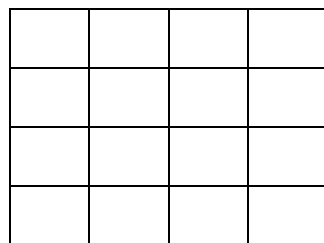
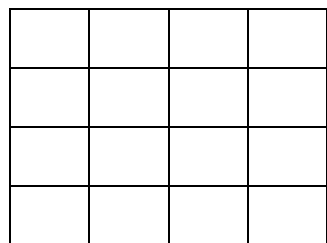
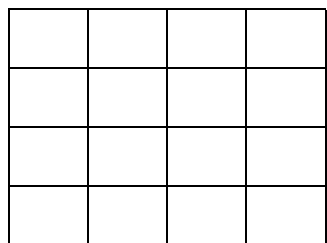
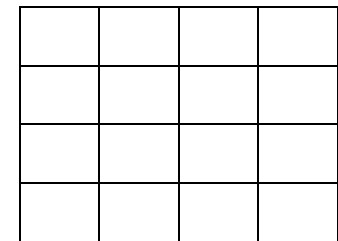
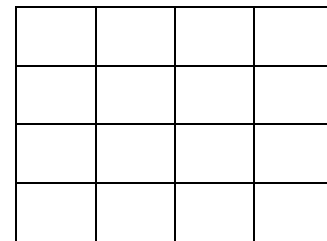
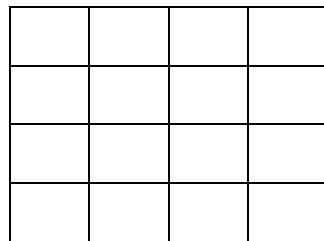
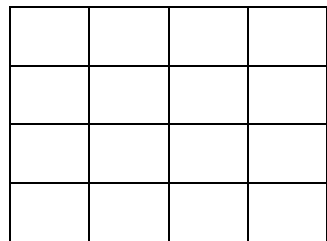
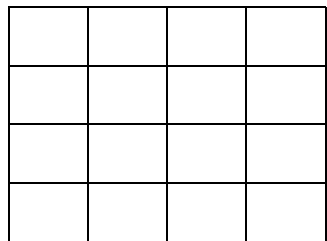
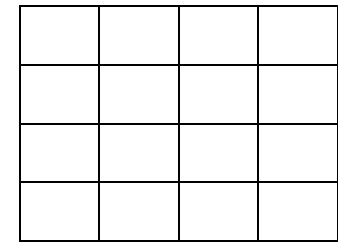
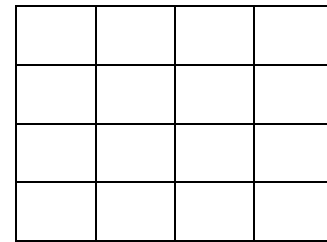
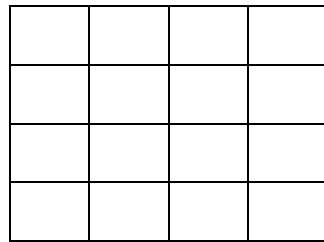
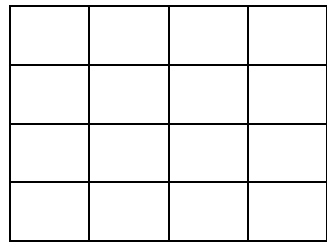
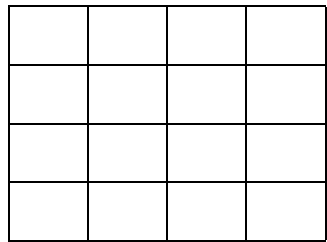
When using the controllable toy, the simpler the device, the better. Many devices remember and accumulate a sequence of instructions, which can be confusing at this stage, so it is important to *clear all* previous instructions before each new instruction is given.



Hundreds	Tens	Ones

## SEEING SQUARES

Cut into separate grids – 1 for each child



# WORKSHEETS FOR LOWER PRIMARY

## 100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

## COOK'S CHERRY SHORTCAKES

*Cook's cherry shortcakes (for ten children)*  
250 g plain flour  
65 g butter  
25 g castor sugar  
150 ml milk  
2 eggs  
140 ml whipped cream  
500 g cherry pie filling

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250 g plain flour  
65 g butter  
25 g castor sugar  
150 ml milk  
2 eggs  
140 ml whipped cream  
500 g cherry pie filling

Colour:		
	in every	
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## ODDS AND EVENS

Odd	Even

Odd	Even

## SIMPLE BATTLESHIPS

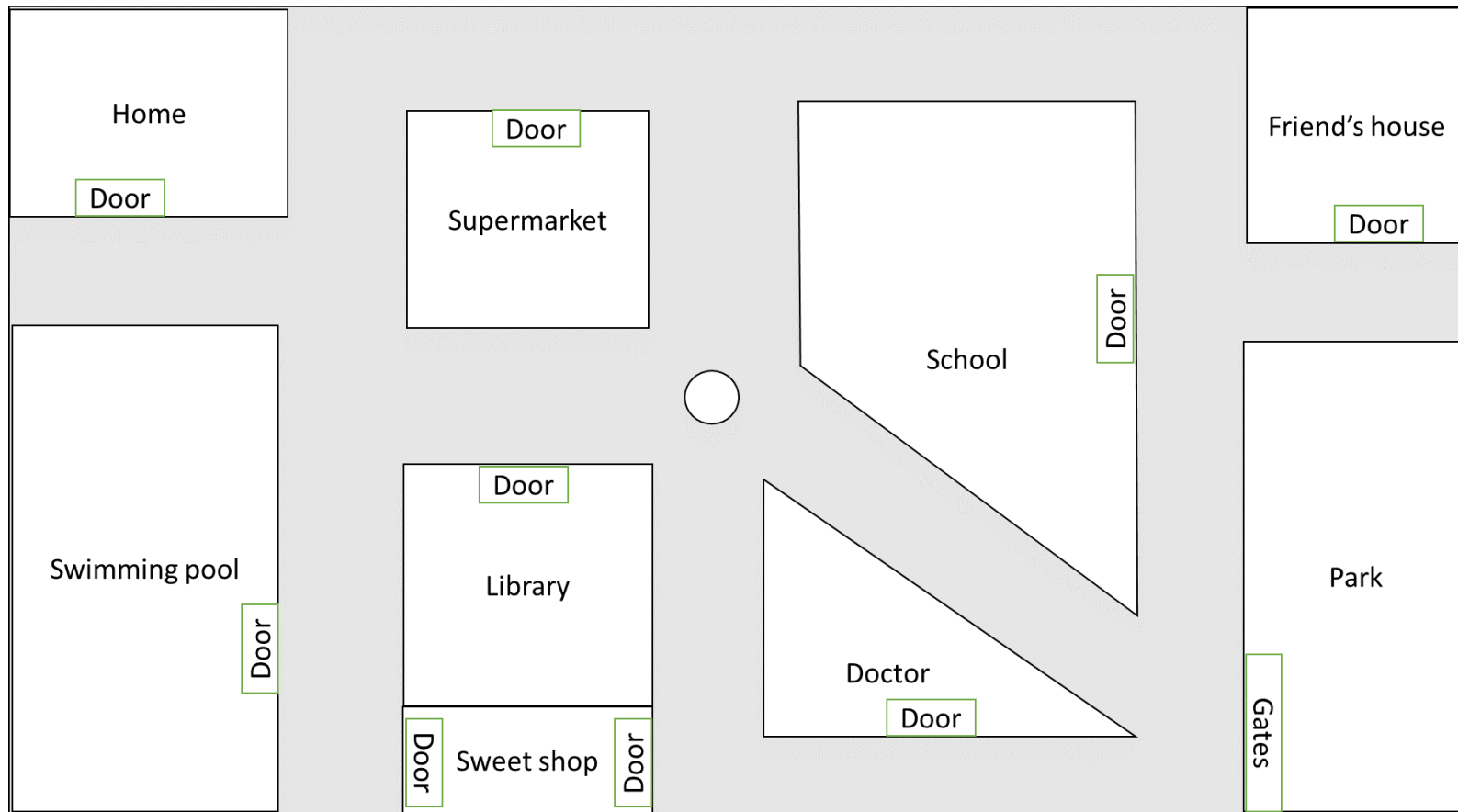
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

## ROBOTS



## SHAPE SORTER


# WORKSHEETS FOR LOWER PRIMARY

## TRAFFIC SURVEY

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>