

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

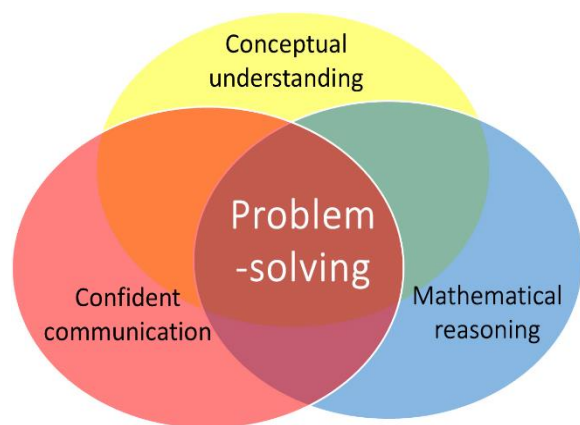
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery*, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*. More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

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24. Transformations and Symmetry

Developing a practical understanding of reflection as an inverse of a shape or movement.

Use the vocabulary: *reflect, reflection, mirror.*

When we look in a mirror the degree of symmetry in our bodies often fools us into thinking that this is the view the world has of us, when of course it is a reflection of that. Someone with a visible asymmetrical feature, such as a birthmark, will unconsciously believe that it appears to everyone on the same side of the face as it appears to them in the mirror. This activity is to help children get used to the idea of reflection being an inverse. It can be carried out as a dance activity during a P.E. session.

Dancing reflections Children work in pairs. The teacher may optionally introduce a piece of music or a poem as a stimulus for movement.

Begin with the whole class standing facing you. Tell the children that they are to pretend to be your reflection in a mirror. As you (slowly) move a part of your body, even simply to wink an eye, ask the children to make the move that you would see in the mirror. Throw in some occasional quicker moves and see if the children remember to reflect with correct body part.

Then ask the children to work in pairs.

Emily leads a series of movements, and Luke endeavours to reflect each movement that Emily makes at the same time. Then they swap over.

With the teacher playing the music (or reciting the poem) as a stimulus, if possible, Luke and Emily make up a sequence of moves (four is sufficient), in which they alternate the lead and the reflection between them with each move. Ask them to practise this so that they make their moves as perfectly reflected as possible.

The pairs perform their dances to the rest of the class. To speed this up, the pair can perform in sets, and ask different parts of the class to watch specific pairs' performances.

Discuss the performances, who led and who reflected? Could the observers see or was the performance too good to identify that? How could the reflections be improved?

During the paired work, it becomes very interesting if the children turn so that they cannot see their partner! Suggest that they include only moves that enable them to see each other.

Challenge higher attaining children to make some of their moves very subtle, just a little turn of the head, or even extending a finger.

When looking at half of a shape reflected in a mirror, many children will say that the two halves they see '*are the same*', rather than recognise that every feature of one half has an ***inverse orientation and position*** in the reflection.

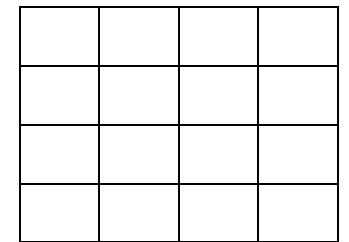
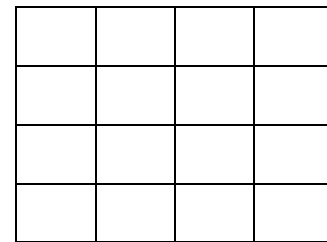
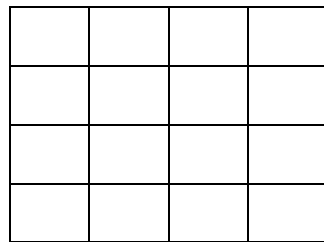
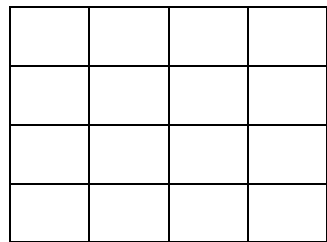
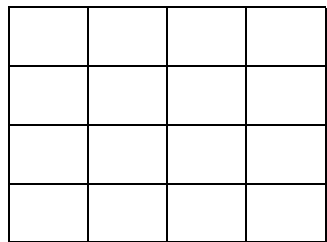
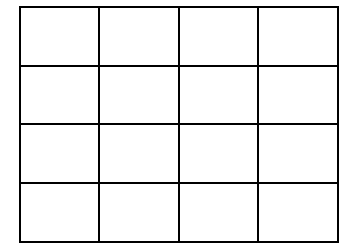
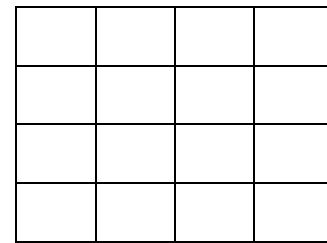
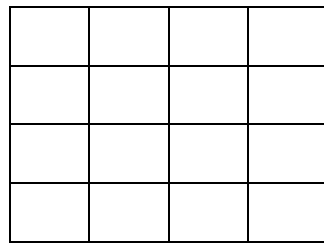
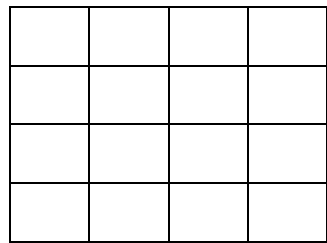
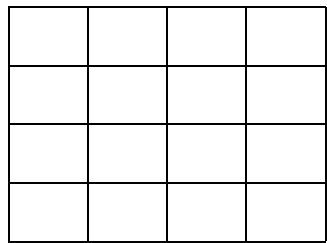
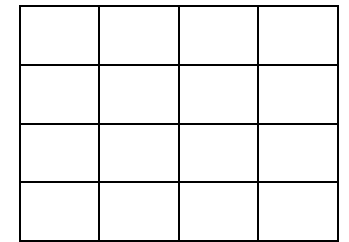
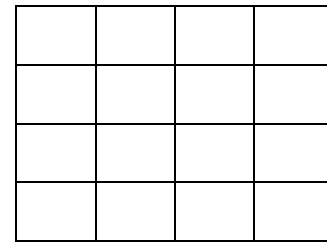
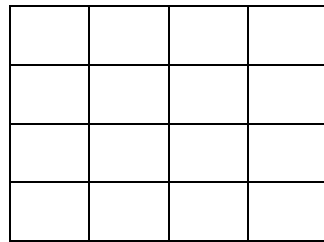
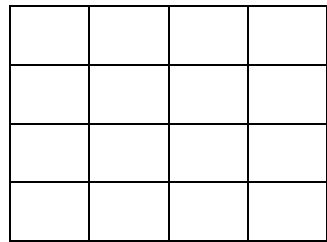
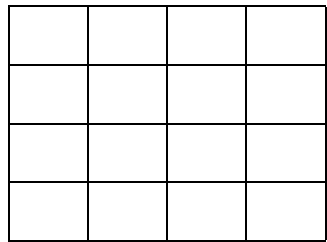
Do the children understand that a ***reflection*** is an inverse orientation of a shape or a movement?

Does the leader's *left* hand appear 'in the mirror' as the reflector's *right* hand, and so on?

Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



WORKSHEETS FOR LOWER PRIMARY

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Colour:		
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ODDS AND EVENS

Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

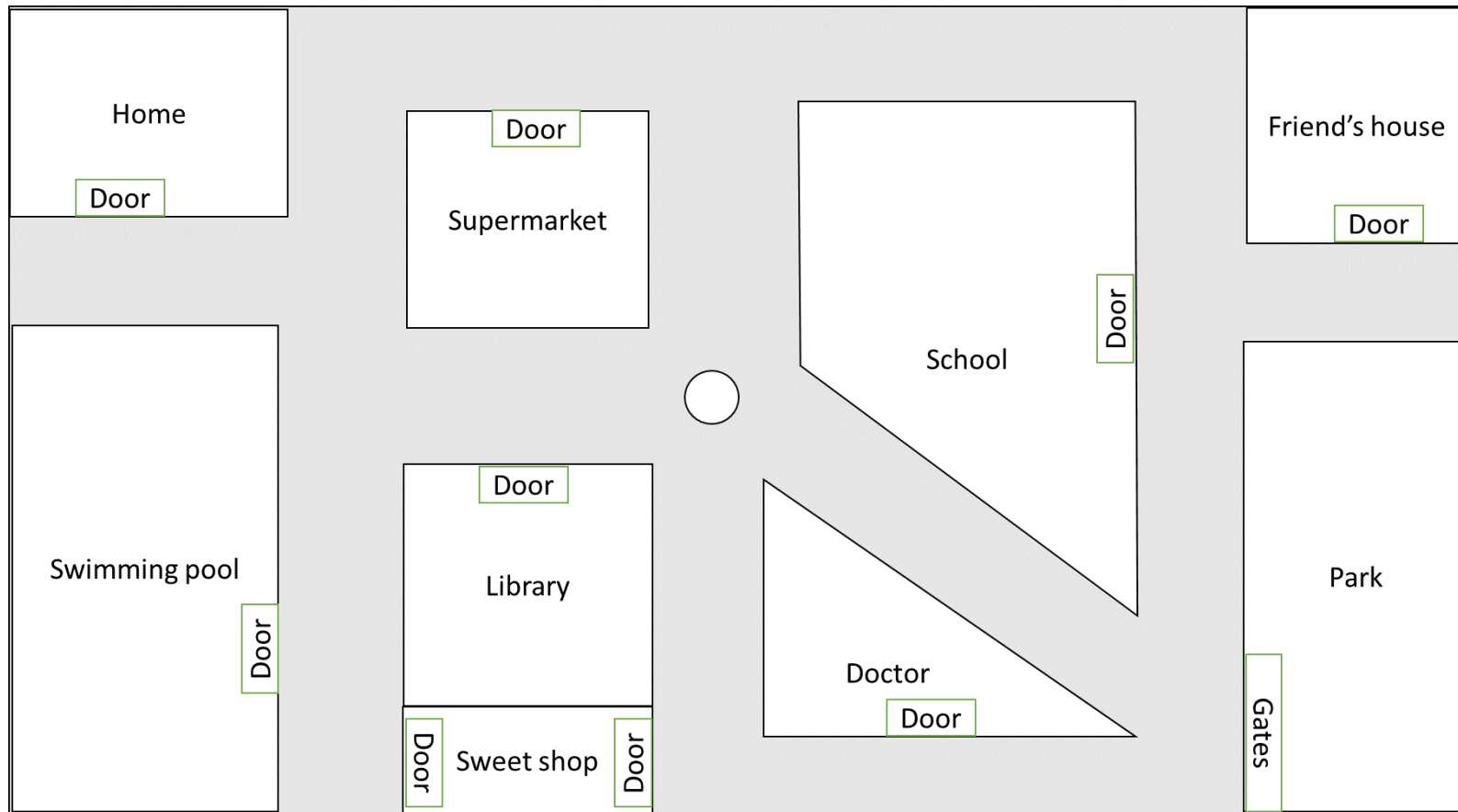
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

ROBOTS



SHAPE SORTER

WORKSHEETS FOR LOWER PRIMARY

TRAFFIC SURVEY

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>