PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

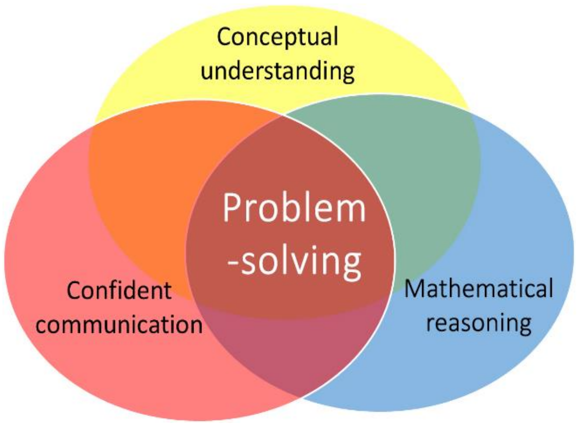
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia,including Bloom's* theories of ***mastery***, the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

Hundreds	Tens	Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

GO OFY WAYS TO MAKE 1!

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

CATALOGUE CHANGES

Barry's Bikes Catalogue		Accessories page	
A. LED light set	$ 20.59	F. Bike helmet	$ 14.57
B. Twin mudguards	$ 16.21	G. 'D' lock	$ 16.9
C. Tyre pump	$ 8.34	H. Cycle computer	$ 9.35
D. Rack	$ 23.98	I. Basket	$ 11.93
E. Gel cycle seat	$ 25.89	J. Puncture repair kit	$ 2.49

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SPORTS MATHS

Team: **Event Name:** **Child's Name**										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

Shape	Angles	Sum of angles

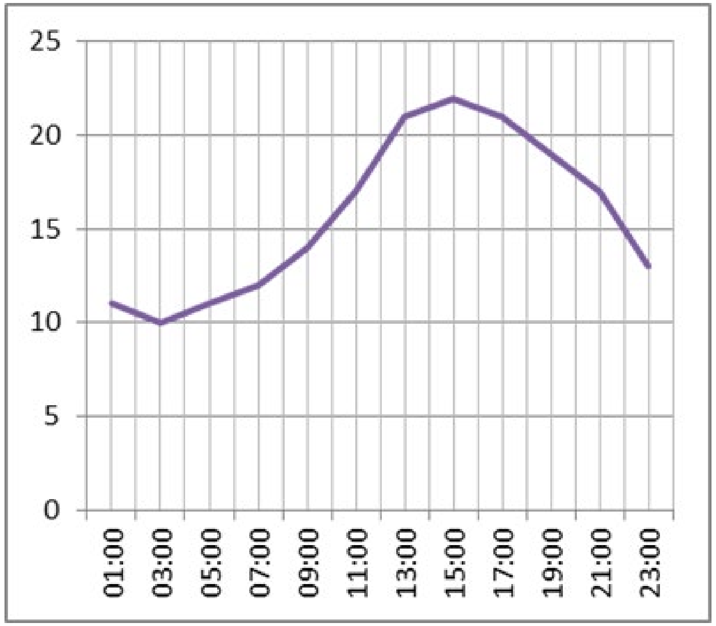
THE LIFE OF PI

object	diameter (cm)	circum-ference (cm)	Possible relationship?

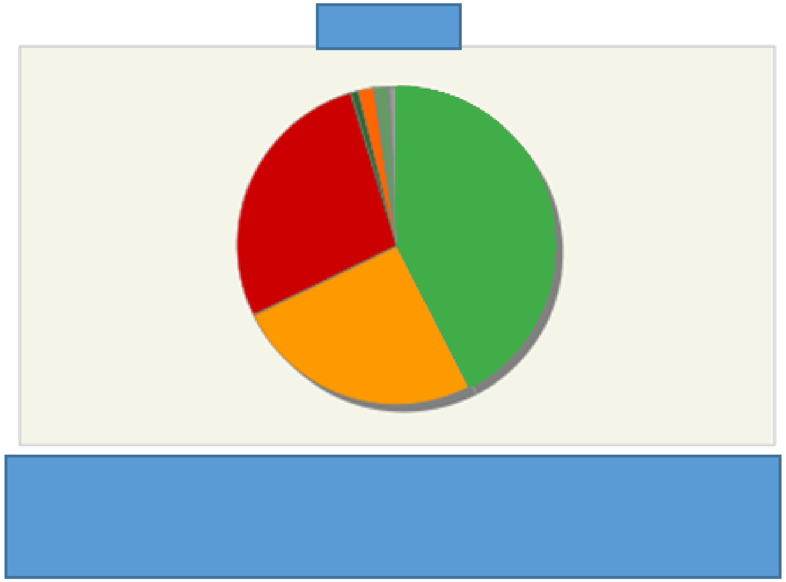
object	diameter (cm)	circum-ference (cm)	Possible relationship?

DATA DETECTIVES

A. Line Graph

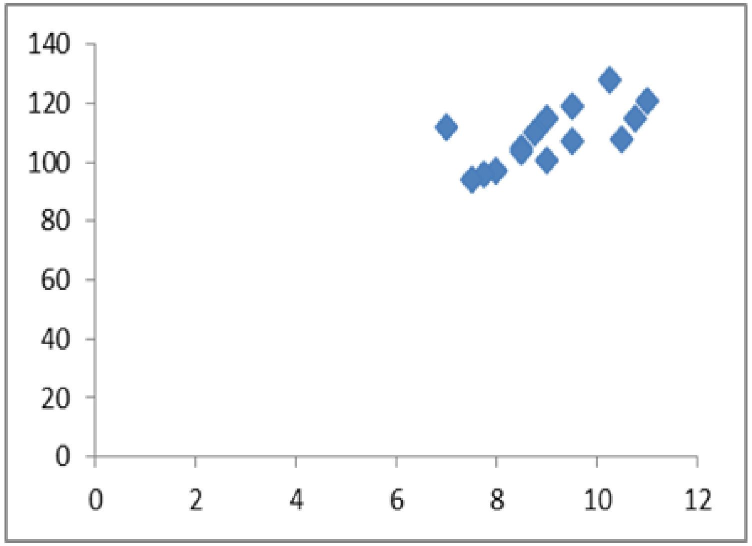


B. Pie Chart

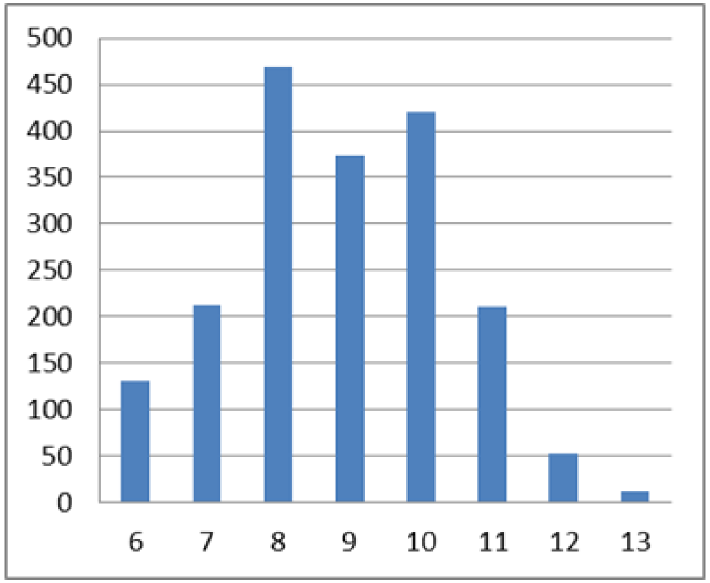


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



TV PROGRAMMES

Programme length	Frequency
Total:	

Programme length	Frequency
Total:	

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NATURE OF THE ACTIVITIES SUGGESTED HERE

In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

25. Classifying Shapes **To discover that there is a special number (π) which is the defining property of a circle.** **To use the vocabulary: *diameter*, *radius*, *circumference*, *ratio*, *π (pi).*** A practical exploration and investigation into the property of a circle that we know as 'π'. For this type of investigation, involving divisions which may produce several decimal places, it is a good idea for the children to use a calculator, to help focus on what is common each calculation.	<u>**The life of Pi**</u> Children explore in pairs. They will need access to: • An *ad hoc* class collection of objects with circular faces: cups, bowls, plates, cylindrical tins, jars, a wall clock, and so on; • Ruler, tape measure and calculator; • Recording sheet (see worksheets). This is best presented to the whole class as a mystery to investigate: What makes a circle a circle? Some children may identify that, for any given circle, the distance from the centre of a circle to its circumference is always the same, no matter where this is measured. A high attaining child may even tell you it is a regular polygon with an infinite number of very small sides! Both of these show great insights into the properties of a circle. Tell the children we are going to investigate to see if there is anything else we can find out. Meena and Charlie work their way through the collection of objects. For each one, they measure and record both the diameter of the circular face, and the perimeter of that circle's circumference. (The latter will be more easily and accurately undertaken using a tape measure.) At the end of their measures the may have a record such as:	Do the children recognise that there is a fixed relationship with the circumference being equal to *'3 and a bit'* times the diameter? Do the children understand why they get a slightly different result for π from each different object? All measurement is approximate, so even their most accurate measuring is imprecise. Do children realise that this number π is unchanging or ***constant***, and is specific to defining a circle?

Later, some children could explore the nature of π as an ***irrational*** (never-ending, non-recurring decimal) ***number***? Ask them to research (online or in books) how precisely π has been calculated so far.

object	diameter (cm)	circum-ference (cm)	Possible relationship?
bowl	17.5	54.5	***54.5 ÷ 17.5 = 3.114***
plate	27.3	86.5	
cup	8.5	27.6	
coaster	11	34.6	

Now ask them to see if they can find any relationship between the measures for each circle. It is appropriate to tell the children they should use calculators for this exploration!

Charlie and Meena find that for every circle they have measured, dividing the circumference by its diameter always gives an answer of '*3 and a bit'* or '3.1-*something*'. Discuss with the children how this is true for every circle, and this special number is the ***ratio*** of any circle's diameter to its circumference which is always '*3 and a bit': 1.* The number is so special that mathematicians have even given it its own symbol (π) and name 'pi' (a letter from the Greek alphabet).

Do they see that as the calculation for the circumference of a circle has been established as ***π × diameter***, and ***diameter*** = ***2 × radius***, we can also calculate the circumference as: ***2 × π × radius***