

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

5-6

NATURE OF THE ACTIVITIES SUGGESTED HERE

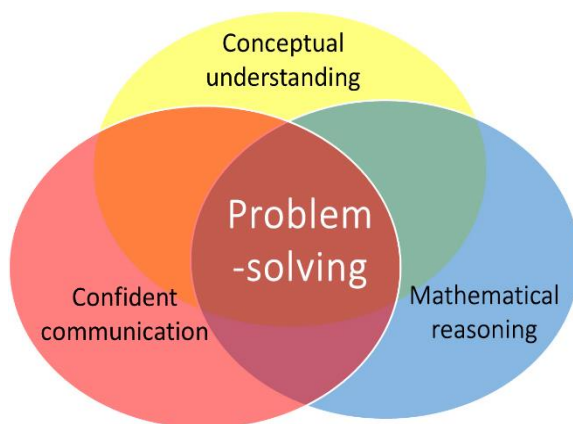
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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NATURE OF THE ACTIVITIES SUGGESTED HERE

In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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26. Handling Data

To interpret the story told by a graph, and draw conclusions from the data.

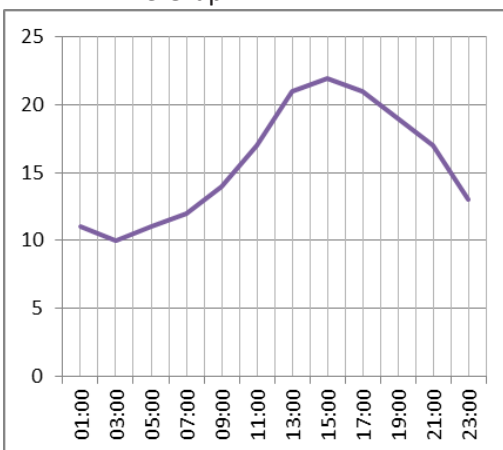
This activity is to challenge children in developing their skills in applying and using the data presented in graphical form.

Data detectives Children work in groups of 3 or 4. They will need:

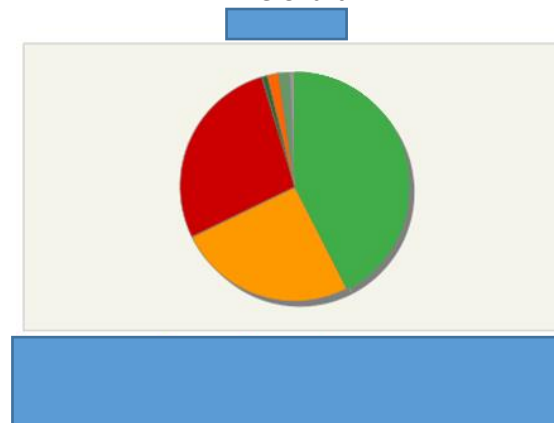
- Selection of different, but incomplete, graphs, so that the information presented may be subject to more than one interpretation or explanation (see worksheets).

Show some simple untitled graphs with some missing labels and/or scales. For example:

A. Line Graph



B. Pie Chart



Can the children in the group *convince* one another of the logical suitability of their explanation and their reasons for this? The children may have more than one valid explanation that fits the data.

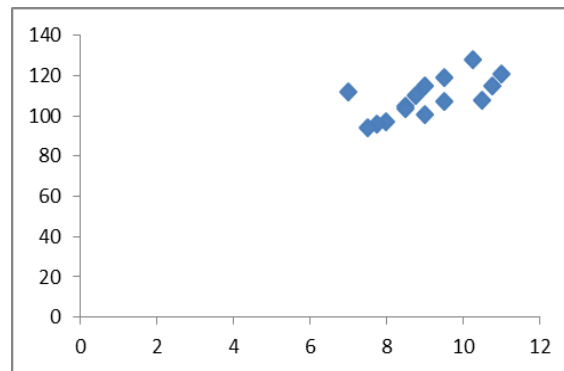
Do the children know the important attributes and uses of each type of graph? When is it appropriate to use one type rather than another? Does this have an impact on the potential explanations for each? For example, only a graph with a continuous relationship between two variables should be represented by a line graph. Of these, showing a variable related directly to time is one of the most common uses.

Can children draw conclusions from the graphs according to their interpretation: what questions could each graph answer? For example:

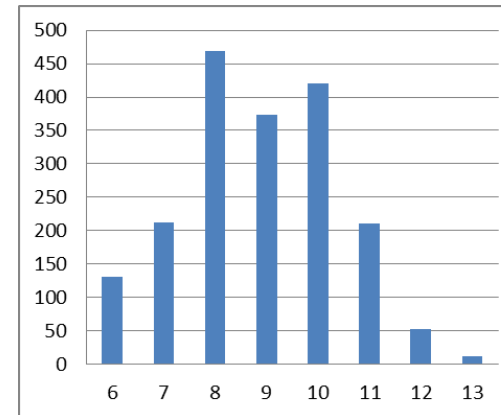
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C. Scatter graph



D. Bar Chart



Meena, Charlie, Alexi and Woljca examine each graph in turn. They discuss and agree between them a convincing story for the data presented in the graph, and complete the missing title, label and scales.

The graphs invite the children come up with potentially more than one convincing story. For the record the originals are:

- Line graph: temperature in $^{\circ}\text{C}$ recorded at different times over a 24-hour period.
- Pie chart: Votes recorded for different parties at a general election: Labour – red, Liberal Party – blue, Australian Greens – green, then smaller parties grouped into other colours.
- Scatter graph: relationship of age (year) to height (cm) for different children in a group.
- Bar chart: Number of pairs (the *frequency*) of men's shoes in different sizes sold by a shoe shop during the same period.

- Why is it warmest mid-afternoon, rather than when the sun is highest in the sky?
- Who won the election? If the pie chart represented the proportion of MPs each party has in Parliament could the governing party be outvoted?
- Do children grow steadily according to age? If so what would be the height/length of a baby when born? Does that make sense?

Hundreds

Tens

Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

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WORKSHEETS FOR UPPER PRIMARY

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

WORKSHEETS FOR UPPER PRIMARY

5-6

GO OFY WAYS TO MAKE 1!

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

CATALOGUE CHANGES

<u>Barry's Bikes Catalogue</u>		<u>Accessories page</u>	
A. LED light set	\$ 20.59	F. Bike helmet	\$ 14.57
B. Twin mudguards	\$ 16.21	G. 'D' lock	\$ 16.9
C. Tyre pump	\$ 8.34	H. Cycle computer	\$ 9.35
D. Rack	\$ 23.98	I. Basket	\$ 11.93
E. Gel cycle seat	\$ 25.89	J. Puncture repair kit	\$ 2.49

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SPORTS MATHS

<i>Team:</i>										
Event Name:										
Child's Name										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

Shape	Angles	Sum of angles

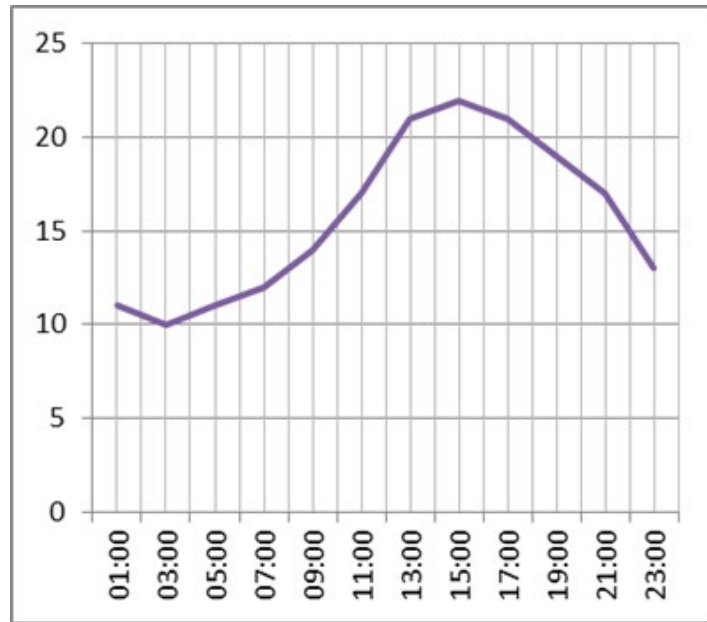
THE LIFE OF PI

<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

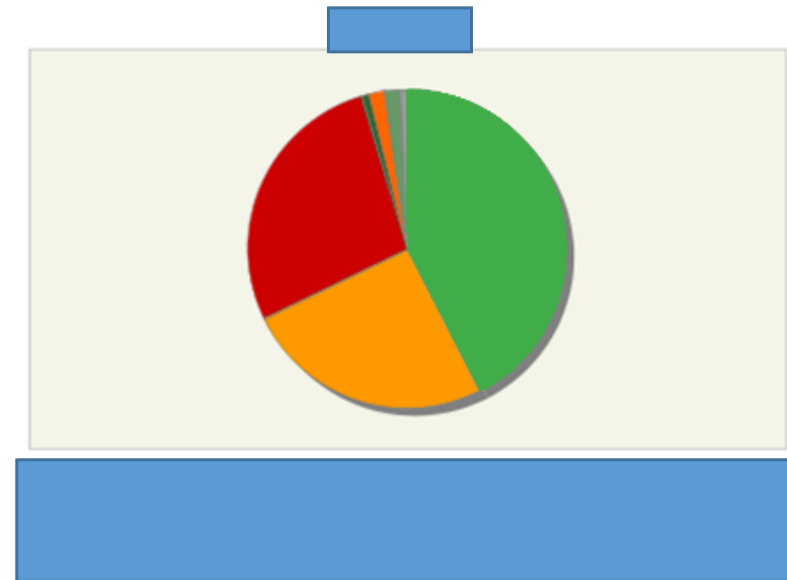
<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

DATA DETECTIVES

A. Line Graph

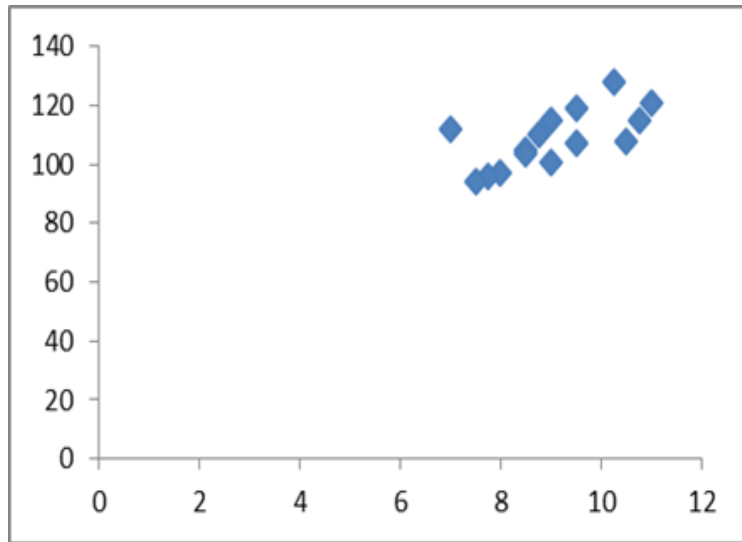


B. Pie Chart

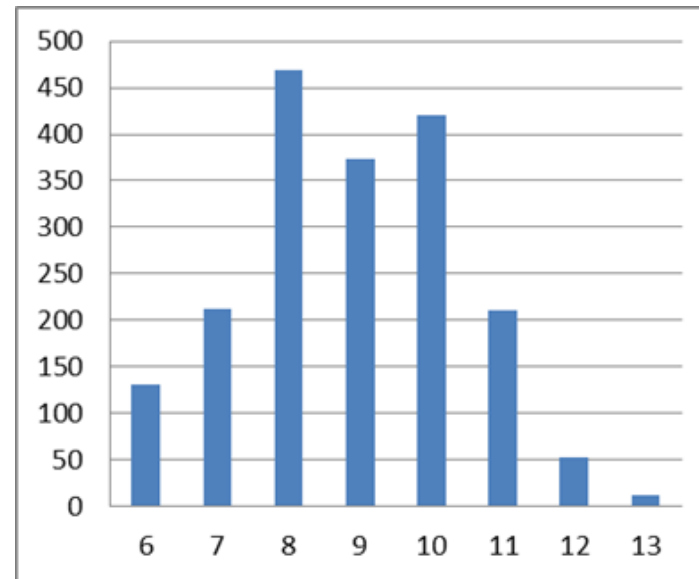


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



WORKSHEETS FOR UPPER PRIMARY

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TV PROGRAMMES

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	