

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

5-6

NATURE OF THE ACTIVITIES SUGGESTED HERE

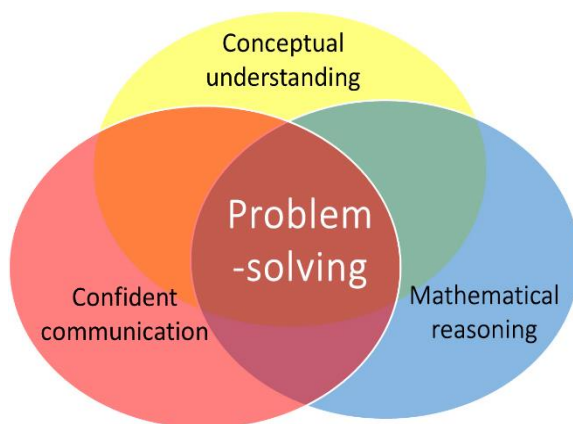
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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<p>27. Comparing Sets of Data</p> <p>To find the <i>mean</i>, <i>median</i> and <i>mode</i> for a set of data.</p> <p>This chapter is mostly focused on the professional needs of teachers, rather than the learning needs of children. However, it is important that by the end of upper primary children understand and use the terms <i>range</i>, <i>minimum</i>, <i>maximum</i> and the <i>mean</i> as a measures of average. So an activity is suggested here on these ideas, but only for children in upper primary.</p> <p>Children will also practically encounter the <i>median</i> and <i>mode</i> as alternative, yet valid interpretations of average from <i>mean</i>. This activity combines data collection and analysis, for which the children will use a TV guide to explore the average</p>	<p>TV programmes Children work in groups of 3 or 4. They will need:</p> <ul style="list-style-type: none">• TV guide or PC/tablet to access this online.• Blank frequency table (see worksheets). <p>Set out the problem that the Controller of a new TV channel wants to know what the average length of a <i>daytime</i> programme (9am–6pm) should be by comparing the averages of programmes on the other channels: ABC, SBS, NITV and Food Network. (Concentrating on these times helps avoid the children reading about potentially inappropriate programmes broadcast at later times in the evening! It may be wise just to photocopy the earlier parts of the evening schedules and distribute these.)</p> <p>Show how to find the difference in minutes between the starting time of one programme and the starting time of the next, using a number of examples.</p> <p>Each child in the group: Meena, Charlie, Alexi and Woljca is to take one of the four channels and calculate the length of time in minutes allocated to each programme to be broadcast on that channel over the same period (say two days). Each child records the raw data in minutes. For example, Charlie collects this data is for ABC for two days between 9am–6pm: 60, 10, 25, 30, 35, 40, 45, 45, 30, 50, 10, 15, 195, 25, 60, 60, 30, 40, 45, 45, 75, 45, 55</p> <p>Charlie then arranges his data in order of increasing programme length: 10, 10, 15, 25, 25, 30, 30, 30, 35, 40, 40, 45, 45, 45, 45, 45, 50, 55, 60, 60, 60, 75, 195</p> <p>Then each child identifies the <i>median</i> (middle data item) programme length for their sample. For Charlie’s data, the median is the 12th item in the order list, a value of 45 minutes.</p> <p>Each child then calculates the <i>mean</i> programme length, by adding all the items in the set and dividing by the total number of items. For Charlie’s data the mean is $1080 \div 23 = 47$ minutes (rounded to the nearest minute).</p> <p>Each child then draws up a <i>frequency table</i>, grouping the data in increasing intervals of, say, 15 minutes, as shown below.</p> <p>Depending on the data collected a larger or smaller interval might be used. They use this the frequency table to identify the group with the highest frequency – which in this case is 31–45 minutes.</p>	<p>Do the children understand why the data has to be sorted in order to identify the <i>median</i> programme length?</p> <p>Do they see how to find the one in the middle of an ordered list? What if there is an even number of items? (Calculate the value halfway between the two middle items.)</p> <p>Do the children realise that it is not necessary to order the data to calculate the <i>mean</i>?</p> <p>Do the children see why a <i>frequency table</i> is needed to calculate the <i>mode</i>?</p> <p>Do they see why grouping the data into intervals is helpful to managing this?</p>
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length of TV programmes using the three measures of average.

In reality, for the use of the **mode** to be valid a much larger set of data should be collected than in the example here.

This group can be identified as the **mode**, in the sense that more programmes have a length in the interval 31–45 minutes than any other interval of 15 minutes.

Programme length	Frequency
1-15 minutes	3
16-30 minutes	5
31-45 minutes	8
46-60 minutes	5
61-75 minutes	1
76-90 minutes	0
91-105 minutes	0
106-120 minutes	0
121-135 minutes	0
136-150 minutes	0
151-165 minutes	0
166-180 minutes	0
180-195 minutes	1
Total	23

As a class discuss the children's findings. What could this data tell the Controller of the new TV channel about people's viewing habits for the different channels?

Extend the challenge by looking at some further channels with different patterns, for example, some of the film channels. What would be appropriate intervals for a frequency chart of those programme lengths?

Hundreds

Tens

Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

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WORKSHEETS FOR UPPER PRIMARY

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

WORKSHEETS FOR UPPER PRIMARY

5-6

GO OFY WAYS TO MAKE 1!

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

CATALOGUE CHANGES

<u>Barry's Bikes Catalogue</u>		<u>Accessories page</u>	
A. LED light set	\$ 20.59	F. Bike helmet	\$ 14.57
B. Twin mudguards	\$ 16.21	G. 'D' lock	\$ 16.9
C. Tyre pump	\$ 8.34	H. Cycle computer	\$ 9.35
D. Rack	\$ 23.98	I. Basket	\$ 11.93
E. Gel cycle seat	\$ 25.89	J. Puncture repair kit	\$ 2.49

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SPORTS MATHS

<i>Team:</i>										
Event Name:										
Child's Name										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

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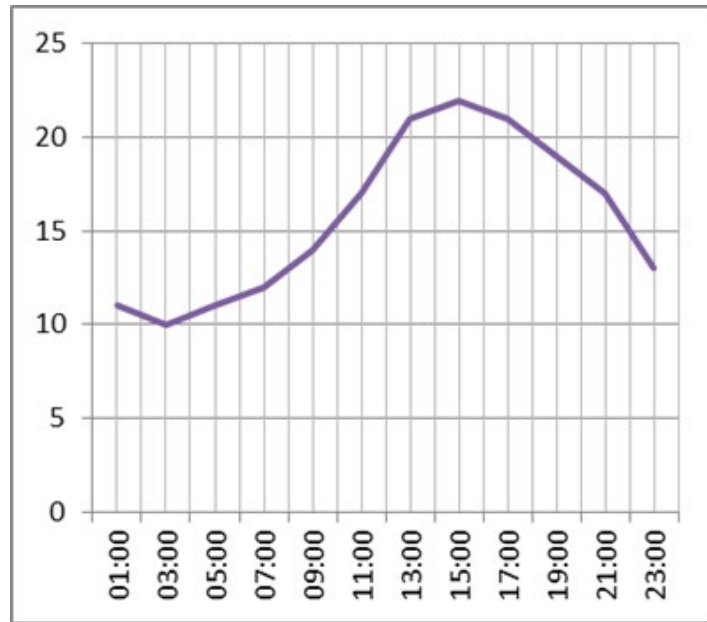
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<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

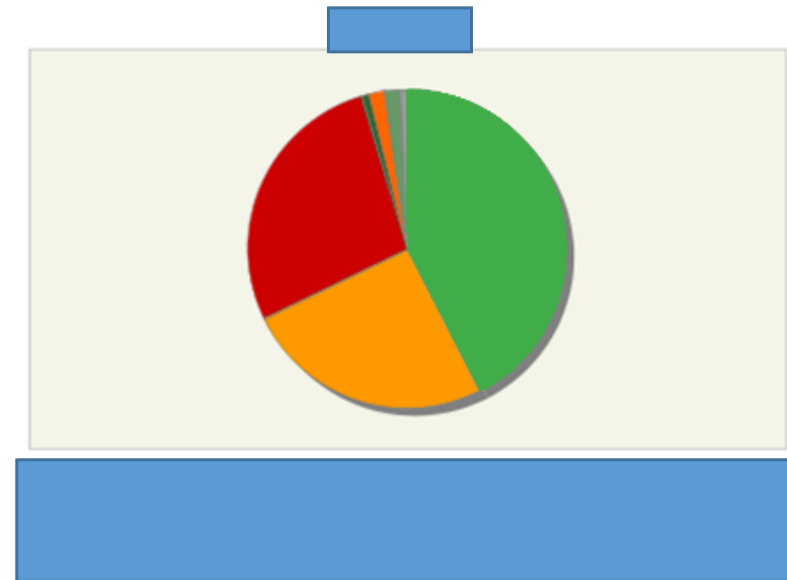
<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

DATA DETECTIVES

A. Line Graph

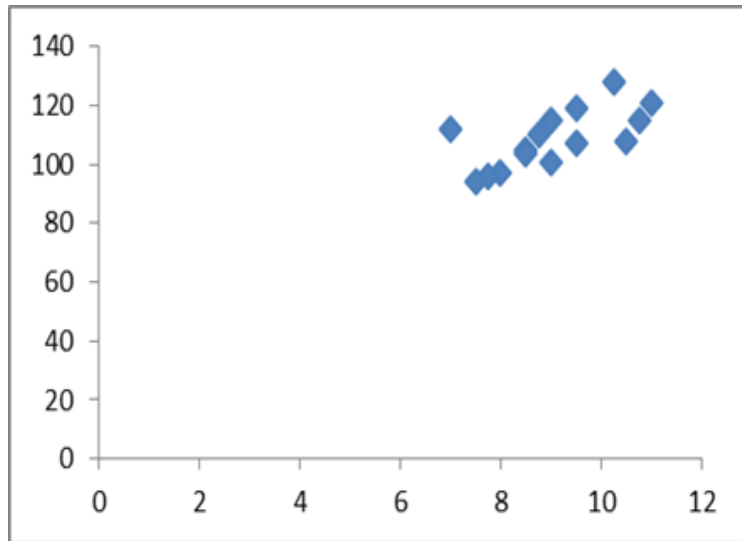


B. Pie Chart



DATA DETECTIVES

C. Scatter graph



D. Bar Chart

