

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

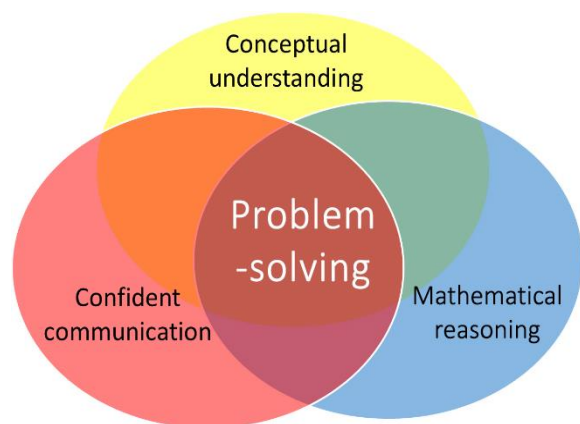
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different approaches of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery*, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*, and the seminal *Cockcroft Report** More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

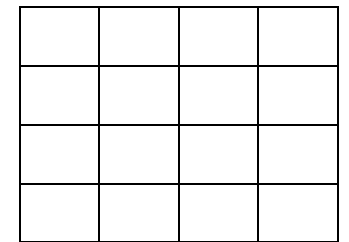
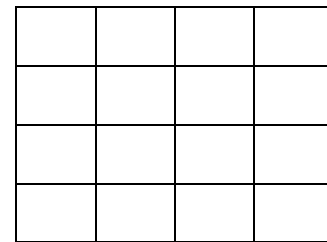
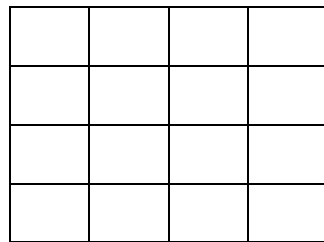
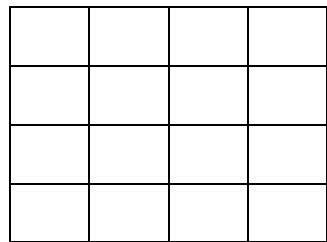
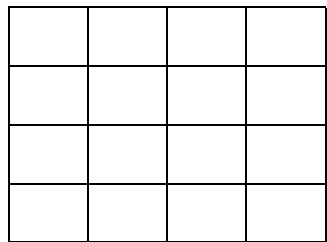
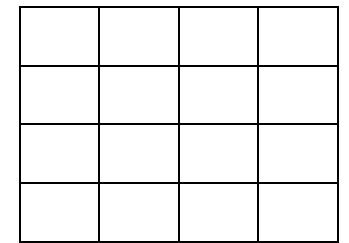
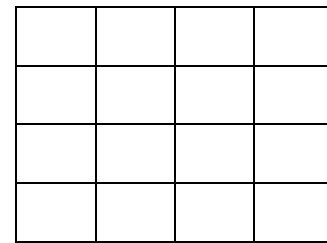
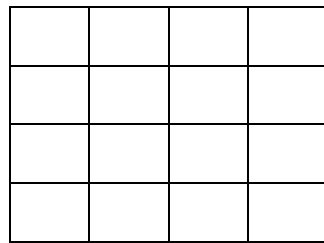
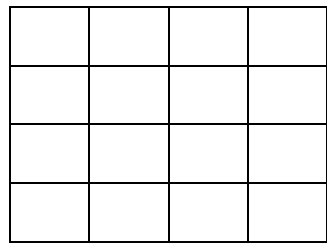
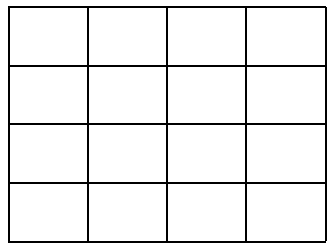
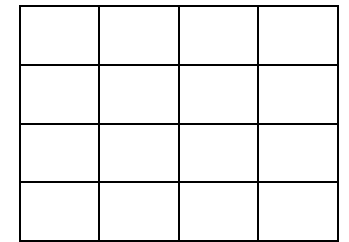
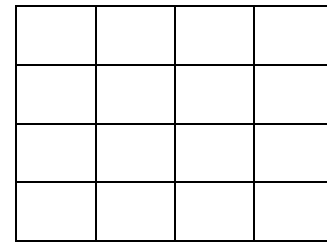
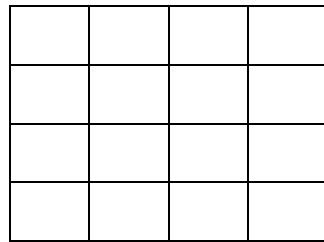
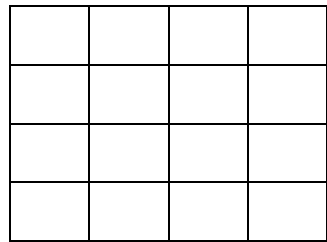
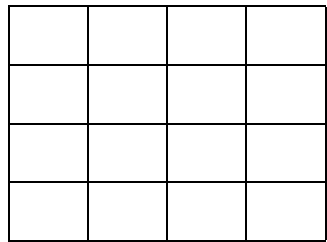
There is also a file of resource sheets used in some of the activities, which may be reproduced freely. However, please include any source information on each copy.

Related chapter, key learning & rationale	Plan for teaching and learning	Crucial points & barriers to understanding
<p>6. Numbers and Place Value</p> <p>Recognise the place value of each digit in a two-digit number (tens, ones).</p> <p>This activity is intended to engage with the same number in four different representations:</p> <ul style="list-style-type: none"> • symbolically and positionally on a number square; • using place value cards to partition into symbolic components in <i>tens</i> and <i>ones</i>; • using <i>base-10 apparatus</i> to partition it in a concrete/physical form; • speaking and hearing the English vocabulary language to describe the number and its place value components. 	<p>Place race The teacher first models the game and then the children play in pairs. They will need:</p> <ul style="list-style-type: none"> • One set of place-value (p.v. or ‘arrow’) cards representing <i>tens</i> and <i>ones</i> for each pair of children; • <i>Base-10</i> or <i>Dienes’ apparatus</i> (<i>tens</i> and <i>ones</i> only); • Place value (p.v.) mat (marked for just <i>tens</i> and <i>ones</i>, or fold back the <i>hundreds</i> on a three-digit p.v. mat); • 100-square (0–99 or 1–100); • Counters in two colours. <p>Luke and Emily agree which colour of counters each will use. They shuffle the p.v. cards and set them out randomly, face down on the table. The <i>base-10</i> apparatus is in a pile the other side of the p.v. cards. Luke chooses any uncovered two-digit number from the hundred square, pointing to and saying the number aloud. The pair then race each other:</p> <ul style="list-style-type: none"> • Luke searches for the correct <i>tens</i> and <i>ones</i> p.v. cards needed to comprise the number and assembles these face up on the table; while ... • Emily grabs the correct numbers of <i>tens</i> and <i>ones base-10</i> pieces and places these in the correct columns on the p.v. mat. <p>The first child to complete their task calls out the number and claims the number by placing their coloured counter over it, then waits for the other to finish (and helps them if necessary). They both check each has the correct cards/numbers of <i>base-10</i> pieces. Then they return the <i>base-10</i> pieces to the pile and shuffle the p.v. cards face down on the table, to begin again. The next time Emily calls the number and searches for the p.v. cards, while Luke grabs the <i>base-10</i> pieces for the p.v. mat. The activity can be simplified by limiting the available range on the number square, or it can be extended to three-digit numbers to make it more challenging.</p>	<p>Do the children match the correct number of <i>base-10</i> pieces to each of the p.v. cards in the number?</p> <p>Do they have to recognise the equivalence of ‘two tens’ and ‘twenty’?</p> <p>Do they speak and interpret correctly the irregular English vocabulary for the numbers 11 to 20?</p> <p>Do the children correct each other if they detect their partner is mistaken?</p>

Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



WORKSHEETS FOR LOWER PRIMARY

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

ODDS AND EVENS

Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

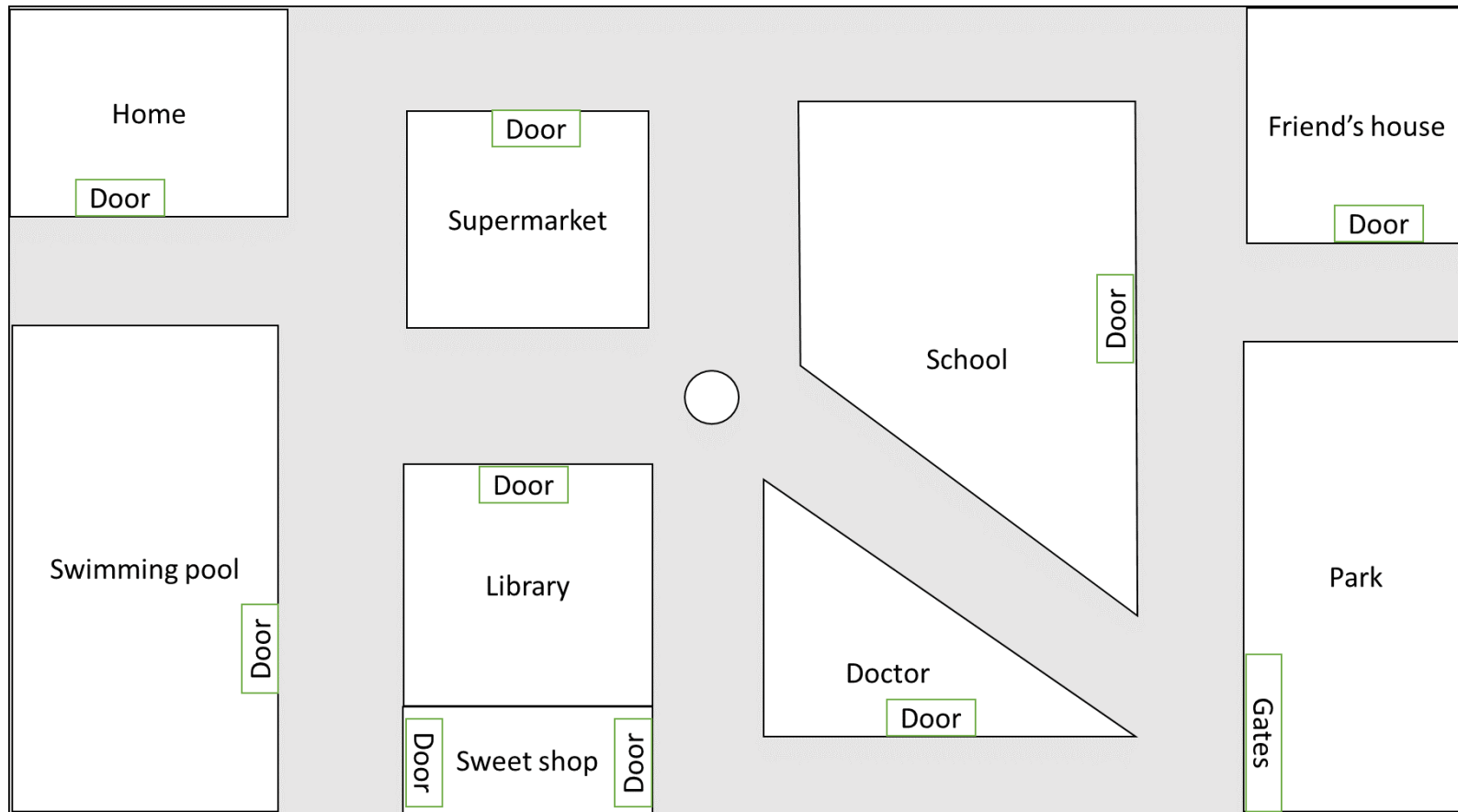
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

ROBOTS



SHAPE SORTER

WORKSHEETS FOR LOWER PRIMARY

TRAFFIC SURVEY

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>