

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

3-4

NATURE OF THE ACTIVITIES SUGGESTED HERE

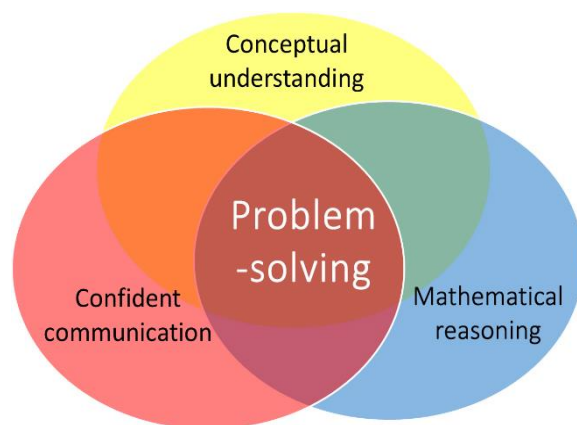
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner* and the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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7. Addition and Subtraction Structures

Use the inverse-of-addition structure to solve a subtraction.

Rounding a number to the nearest 5.

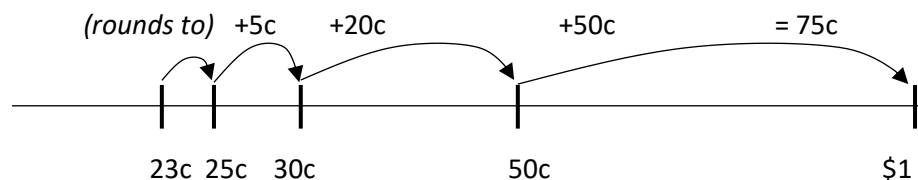
Solve number and practical problems.

In street markets, traders still use the inverse-of-addition structure physically to count out the correct change to give to a customer who tenders a larger denomination coin or note for payment.

Market trader The teacher models the activity, then the children work together in pairs. They will need:

- A small tray of coins of different denominations, up to \$1;
- A number of different objects, for example: book, pencil, eraser, ruler, etc., each labelled within a range of prices up to 99c.

Shelley is a market trader with the items for sale and the tray of coins. Rohan starts with a \$1 coin. Rohan chooses something from Shelley's stall and hands over a \$1 coin. Shelley starts from the price of the item and counts on coins to arrive at the \$1 Rohan offered in payment. For example, Rohan buys the pencil for 23c, so Shelley takes the \$1, and collects the change to give Rohan, writing it on a number line and describing it as she goes: '23c is rounded to 25c; and 5c is 30c; and 20c is 50c; and 50c makes \$1, so your change is 75c'.



Rohan counts the change (75c) and checks it, writing down '\$1 - 25c = 75c'. They swap roles, so that Rohan becomes the trader and Shelley is the customer, and continue to alternate turns.

To simplify the activity, reduce the customer's money, say to 50c. To extend it, find the change up to a \$2 coin or a \$5 note or beyond, or let them buy more than one item at a time.

Do children emphasise the term **adding on** as a way of finding the difference?

Do children recognise the connection between **adding on** to find the change, and the subtraction number sentence?

Do they add and subtract values correctly?

Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					
	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

ROUNDING TO ESTIMATE

<i>The Cosy Café</i>		
<u>Savoury</u>		
Sandwich	\$1.95	Baguette \$2.75
Tortilla wrap	\$1.45	Pizza slice \$2.10
<u>Sweet</u>		
Scone and jam	\$2.10	Cake \$2.90
Shortbread	\$1.25	Tiffin \$1.75
<u>Drinks</u>		
Squash	95c	Juice \$1.35
Tea	\$1.65	Coffee \$2.15

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ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

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PROBABLE PROPORTIONS

<i>Colour</i>	<i>Tally</i>	<i>Total</i>

<i>Colour</i>	<i>Tally</i>	<i>Total</i>