PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

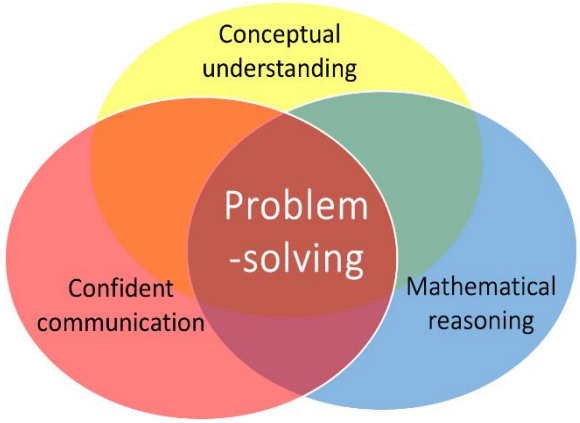
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of ***mastery,*** the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner* and the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

8. Mental Strategies for Addition and Subtraction **Mentally add and subtract numbers with up to three digits.** Not only is this a very effective game for practising mental addition and subtraction, it is a great way to introduce children to the joy of strategic card games and a super vehicle for mathematical thinking.	**Junior Cribbage** Children play in groups of 4. They will need the following resources for each group: • Pack of playing cards; • Counters for scoring points. This is activity is based on the card game *cribbage*, in which players take turns to choose and lay a card, adding the face-value of the card to an accumulating total. The Jack, Queen and King each have a value of 10. Each round, target values of 15 and 31 score points for the player whose card completes those totals, so players need to think about which cards have already been played and what is more helpful to them to lay when it is their turn. For this version, in order to practise quick mental addition of one-digit and tens to three-digit numbers, you can set a whole series of ***targets*** up to a possible maximum of 340 (the sum of all the face values of the cards and 10 for each of the J, Q and K). For example: 15, 31, 40, 52, 63, 79, 88, 94, 106, 115, 131, 140, 152, 163, 179, 188, 194, 206, 215, 231, 240, 252, 263, 279, 288, 294, 306, 315, 331, 340 (30 target numbers per round). Rohan shuffles the cards first and deals out the cards face down one at a time to each player, starting with Shelley on his left, so that each player has 13 cards of unpredicted values. Players pick up and look at their own cards, but must not show them to anyone else. On Rohan's left, Shelley begins by laying one of her cards face-up. She states the value of the first card and each player lays a card in turn, always saying the accumulated total so far. For example: Shelley first lays a queen, and states the total so far: '10'. Next to her, Harry lays a 5, and says the total so far: 'That makes 15'. As this is also a target number in our list, Harry scores a point and takes a counter. Next to him, Mia lays a 9, and calls the new total, e.g. '15 add	Can the children start with an existing accumulated subtotal and identify the correct value of the card needed by which they can augment/increase it correctly for the next target number? Do they do this by recognising the inverse relationship between addition and subtraction subtracting from the *next* target to identify the card they need to lay? Do they add and subtract values correctly? What strategy do they take if they cannot make the next target number? Do they choose to exceed it, so as to reduce the points available to other players?

	9 is equal to 24'. As the new total is not a target number, Mia does not take a counter. Rohan lays a 7: 'That makes 31 and a point for me'. Rohan takes a counter and Shelley's turn arrives again, e.g. she lays a 9: 'and that makes 40', so Shelley takes a counter and the next target number is 52 when Harry lays his next card. The children continue to play in turn, scoring points for target numbers, until 340 is reached. If at any point a player lays a card that makes the total greater than the next target number, play simply continues to the next target number without anyone gaining a point. Children need to check and agree the new total is correct each time a card is played. The activity can be simplified by reducing the number of cards (and the maximum total, for example, take out all picture cards to limit to 220). It can be extended by assigning higher values for the picture cards, e.g. J = 11, Q = 12, K = 13. It can be made more challenging by allowing each player to lay up to two cards at a time and to add their sum to the accumulated total.	

Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					
	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

The Cosy Café			
Savoury			
Sandwich	$1.95	Baguette	$2.75
Tortilla wrap	$1.45	Pizza slice	$2.10
Sweet			
Scone and jam	$2.10	Cake	$2.90
Shortbread	$1.25	Tiffin	$1.75
Drinks			
Squash	95c	Juice	$1.35
Tea	$1.65	Coffee	$2.15

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ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

PROBABLE PROPORTIONS

Colour	*Tally*	*Total*

Colour	*Tally*	*Total*